Queen Mary and Westfield College
University of London

MAS200 ACTUARIAL STATISTICS (B.Sc. examination by course units)

Duration: 2 hours
Date and time: MOCK EXAM PAPER

You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best THREE questions will be counted.

You are permitted to use an electronic calculator in this examination but you may not use any preprogrammed or graphical features which it may have. Please state on your answer book the name and type of machine used.

Question 1. (*30 marks*)

Give your answer in parts (a) – (d) to 2 decimal points.

(a) (*4 marks*) The nominal rate of discount convertible quarterly is 7% per annum. Find the effective annual rate of discount .

(b) (*4 marks*) For a constant force of interest $\delta = 0.12$, find the amount of interest to be paid in arrears for use of 1 unit of money over one month.

(c) (*6 marks*) A Sony playstation with a cash price of £189.99 can be purchased for 26 equal monthly repayments with the first repayment being due at the moment of purchase. Find the amount of each monthly repayment if the APR charged is 26.7%.

(d) (*6 marks*) Show that $\ddot{a}^{(p)}_{\overline{n}|} = \dfrac{i}{d^{(p)}} a_{\overline{n}|}$

(e) (*10 marks*) A loan of £10,000 was repaid quarterly in arrears for 10 years. The repayment terms provided that at the end of the first 5 years the amount of the quarterly repayment would be increased by £100. The amount of the annuity was calculated on the basis of the nominal rate of interest 12% per annum convertible quarterly.

Find the initial amount of the quarterly repayment.

Question 2. (*30 marks*)

(a) (*3 marks*) Define the force of mortality $\mu(x)$ in terms of the survival function. Is it possible for $\mu(x)$ to be greater than 1? If not explain why, if yes give an example.

(b) (*3 marks*) Define the probability ${}_tq_x$. Express it in terms of the survival function. Find ${}_tq_x$ for De Moivre's Law of Mortality $s(x) = k(\omega - x)$, $0 \le x \le \omega$.

(c) (*4 marks*) Show that the survival probability ${}_tp_x$ does not depend on age x if the age-at-death random variable X has an exponential distribution.

(d) (*6 marks*) Let $K(x)$ be the integer part of $T(x)$, the time-until-death random variable. Express the probability mass function of $K(x)$ in terms of the survival probabilities ${}_kp_x$. Define the curtate expectation of life, e_x. Show that $\mathsf{e}_x = \sum_{k=1}^{\infty} {}_kp_x$.

(e) (*6 marks*) Show that

$$p_x = \frac{\mathsf{e}_x}{1 + \mathsf{e}_{x+1}}.$$

(f) (*8 marks*) Prove that $E(X|x_1 < X < x_2)$, the expected age at death of those lives who die between age x_1 and age x_2 ($x_1 < x_2$) is given by the integral

$$\int_{x_1}^{x_2} \frac{xs(x)\mu(x)}{s(x_1) - s(x_2)}\, dt,$$

where $s(x)$ is the survival function and $\mu(x)$ is the force of mortality.

Question 3. (*30 marks*)

(a) A population is subject to mortality according to English Life Table No. 12 – Males appended to the end of this examination paper.

(*3 marks*) Find the probability that a man aged 50 will survive for at least 10 years. Give your answer to 3 significant digits.

(*3 marks*) What is the probability a man aged 60 will die between ages 65 and 70? Give your answer to 3 significant digits.

(*3 marks*) In a group of 1000 newborns, find the expected number of those who will die aged 60 and 61 last birthday. Give your answer rounded to the nearest integer.

(*6 marks*) A man aged 50 has just retired because of ill health. His future mortality is that of E.L.T. No. 12 – Males except for the first year when his chance of surviving the year is half of that of the value in the table. Find the complete expectation of his further life on retirement through ill health. Give your answer to 2 decimal places.

(b) (*4 marks*) Show that under assumption of uniform distribution of deaths within each year of age, the following relation holds for all integer x and fractional t:

$${}_tp_x\mu(x+t) = q_x.$$

(c) (*3 marks*) Let z_1 be the present value of unit death benefit under a whole-life assurance policy with the death benefit payable immediately on the moment of death. Express z_1 in terms of $T(x)$, the time-until-death for the life to be assured.

(d) (*3 marks*) Let z_2 be the present value of unit death benefit under a whole-life assurance policy with the death benefit payable at the end of the year of death. Express z_2 in terms of $K(x)$, the number of complete years to be lived by the life age x to be assured.

(e) (*5 marks*) The expected present values of z_1 defined in (c) and z_2 defined in (d) are denoted by the symbols $\bar{A}_x$ and A_x respectively. Using (b), show that under the assumption of uniform distribution of deaths

$$\bar{A}_x = \frac{i}{\delta} A_x,$$

where i is the effective annual rate of interest and δ is the force of mortality.

Question 4. (*30 marks*)

Answer (a) – (c) on the basis of the life assurance table A1967–70 with 4% interest, appended to the end of this examination paper. Use select value for age 40. Give your answers to 2 decimal places.

(a) (*3 marks*) A whole-life annuity-due, issued to a life aged 40, pays 1 unit of money annually. Find the expected present value of the annuity.

(b) (*3 marks*) A whole-life assurance, issued to a life aged 40, has a sum assured of £100,000. The sum assured is payable at the end of the year of death. Find the expected present value of the benefit payment.

(c) (*3 marks*) Premiums for the whole-life assurance in (b) are paid annually in advance for life. Using the equivalence principle and the results obtained in (a) and (b), calculate the annual premium (net value).

(d) (*11 marks*) A life company withdraws benefit payments from an investment fund earning interest of 8% per annum effectively. Suppose that the company has just sold a block of 100 whole-life assurances with a sum assured of £50,000 payable immediately on the moment of death.

Assuming a constant force of mortality $\mu = 0.1$, calculate the minimum amount to be invested by the company at the present time so that the probability is approximately 0.95 that sufficient funds will be on hand to withdraw the benefit payments on death of each life assured; use a normal approximation for the total present value of the benefit payments.

(If N has the standard normal distribution then $P(N < 1.645) = 0.95$.)

(e) (*10 marks*) Consider a pure endowment policy with a term of n years and unit benefit. Let z be the present value of its benefit payment. Assuming a constant force of mortality μ and a constant force of interest δ write down an expression, in terms of μ and δ, for z regarded as a random variable and obtain its probability density function.

Question 5. (*30 marks*)

(a) The following Leslie matrix applies to a certain population of beetles

$$\begin{pmatrix} 0 & 0 & 6 \\ \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \end{pmatrix}$$

The female beetles in the population live 3 years only and propagate in their third year of life.

(*5 marks*) What is the probability that a newborn female beetle will survive to the 3rd year of its life?

(*13 marks*) Assuming a thousand females in each age group at time $t = 0$, how many females will be in the age group at times $t = 1, 2, 3, 15$?

(b) (*12 marks*) Females in a population act independently. Initially the population contains N_0 females. In any small time interval δt, the probability that an individual dies is $\mu\delta t + o(\delta t)$ and the probability that it gives birth is $\beta\delta t + o(\delta t)$. The probability of multiple births and deaths in any small interval δt is $o(\delta t)$.

Show that the probability, $P_n(t)$, that at time t there are n females in the population satisfies the equation

$$\frac{d}{dt}P_n(t) = \mu(n+1)P_{n+1}(t) - (\beta+\mu)nP_n(t) + \beta(n-1)P_{n-1}(t).$$

End of examination questions. A list of selected formulae and Tables ELT-12 and A1967-70 follow.

Present values of annuities-certain:

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{1-v} \qquad a_{\overline{n}|} = v\,\ddot{a}_{\overline{n}|}$$

$$\ddot{a}^{(p)}_{\overline{n}|} = \frac{1}{p}\left[\frac{1-v^n}{1-v^{\frac{1}{p}}}\right] \qquad a^{(p)}_{\overline{n}|} = v^{\frac{1}{p}}\,\ddot{a}^{(p)}_{\overline{n}|}.$$

Expected present values of life annuities:

$$\ddot{a}_x = \frac{N_x}{D_x} \qquad \ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x} \qquad a_x = \ddot{a}_x - 1$$

$$\ddot{a}^{(p)}_x \simeq \ddot{a}_x - \frac{p-1}{2p} \qquad a^{(p)}_x \simeq a_x + \frac{p-1}{2p} \qquad a^{(p)}_x = \ddot{a}^{(p)}_x - \frac{1}{p}$$

Conversion relationships:

$$\bar{A}_x = 1 - \delta\bar{a}_x$$

$$A_x = 1 - d\,\ddot{a}_x$$

$$A^{(p)}_x = 1 - d^{(p)}\,\ddot{a}^{(p)}_x$$

$$A^{(p)}_{x:\overline{n}|} = 1 - d^{(p)}\,\ddot{a}^{(p)}_{x:\overline{n}|}$$