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MAS200

Actuarial Statistics

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Cash flows

Discrete cash flows.

The present value (i.e. at time t = 0) of payments of $c_1, c_2, ..., c_n$ due at times $t_1, t_2, ..., t_n$ in the future is

$$c_1 v^{t_1} + c_2 v^{t_2} + \ldots + c_n v^{t_n} = \sum_{j=1}^n c_j v^{t_j}$$
(1)

Continuously payable cash flows (payment streams).

We say money is paid continuously in time interval $[t_1, t_2]$ if

$$\rho(t) = \lim_{h \to 0} \frac{\text{amount paid over time } [t, t+h]}{h}$$

is a continuous function of time *t* and $\rho(t) > 0$ for all $t_1 < t < t_2$.

The function $\rho(t)$ is called the *rate of payment* at time *t*.

The present value (i.e. at time t = 0) of a continuous payment at rate $\rho(t)$ over a time interval $[t_1, t_2]$ in the future is

$$\int_{t_1}^{t_2} v^t \rho(t) dt.$$
 (2)

Notice that

$$\int_{t_1}^{t_2} v^t \rho(t) dt = \int_{t_1}^{t_2} e^{-t\delta} \rho(t) dt, \quad \text{as } v = (1+i)^{-1} = e^{-\delta}$$

The present value of continuous payment over time interval [0,1], at a constant rate $\rho(t) = \rho$ is

$$\rho \int_0^t e^{-t\delta} dt = iv \frac{\rho}{\delta}.$$

In particular, if the rate of payment equals the force of interest, i.e. $\rho = \delta$, then the present value of such continuous payment is *iv*, see the table "Five equivalent ways of paying interest" below.

Turn over

We say that two payments are *equivalent* if their present values coincide.



Five equivalent ways of paying interest on a loan of 1 unit of money over [0,1]*:*

Notice that

- the present value of continuous payment at constant rate $\rho(t) = \delta$ over [0,1] (entry (5) in the table above) is $iv \frac{\rho}{\delta}\Big|_{\rho=\delta} = iv$,
- the present value of a single payment of *i* at time t = 1 (entry (4) in the table) is *iv*,
- the present value of a single payment of d at time t = 0 (entry (1) in the table) is d and d = iv.

Therefore $(1) \Leftrightarrow (5) \Leftrightarrow (4)$.

Exercise: obtain $(3) \Leftrightarrow (4) \Leftrightarrow (2)$.

Equation of value

Example. 1.

A borrower is under an obligation to repay a bank \pounds 6280 in four years' time, \pounds 8460 in seven years' time and \pounds 7350 in thirteen years' time. As part of a review of his future commitments the borrower now offers either

(a) to discharge his liability for these three debts by making an appropriate single payment five years from now; or

(b) to repay the total amount owed (i.e. £22090) in a single payment at an appropriate time.

On the basis of an constant rate of interest 8% per annum effectively find the appropriate single payment if offer (a) is accepted by bank, and the appropriate time to repay the entire indebtedness if offer (b) is accepted.