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MAS200 Actuarial Statistics

Pure Endowment: An Example Spring 2001

Worked example. John, age 30, is taking out a pure endowment policy with a term of 10 years. Under this policy John will receive £1,000 on his survival to the age of 40.

On the basis of A1967-70 select, calculate the expectation and standard deviation of the present value of the benefit payment to John at 4% interest.

Ignoring expenses, how much should John expect to pay for his policy?

Solution. The P.V. of 1,000 due on John's survival the following 10 years is

$$z = \begin{cases} 1000v^{10}, & \text{if John survives to the age of 40} \\ 0, & \text{otherwise.} \end{cases}$$

The probability that John survives to the age of 40 is ${}_{10}p_{[30]}$. Therefore, $z = 1000v^{10}Y$, where $Y \sim \text{Bernoulli}({}_{10}p_{[30]})$.

From this, the expected present value of the benefit payment is

$$E(z) = E(1000v^{10}Y) = 1000v^{10}E(Y) = 1000v^{10}{}_{10}p_{[30]}$$

and the variance of the present value is

$$\text{var}\,(z) = \text{var}\,(1000v^{10}Y) = (1000v^{10})^2\text{var}\,(Y) = (1000v^{10})^2{}_{10}p_{[30]}(1 - {}_{10}p_{[30]}),$$

so that the standard deviation is

$$\text{st.dev}\,(z) = 1000v^{10}\sqrt{{}_{10}p_{[30]}(1 - {}_{10}p_{[30]})}.$$

Since

$$A_{[30]:\overset{\;1}{\overline{10}|}} = v^{10}{}_{10}p_{[30]} = v^{10}\frac{l_{[30]+10}}{l_{[30]}} = \frac{v^{30+10}l_{[30]+10}}{v^{30}l_{[30]}} = \frac{D_{[30]+10}}{D_{[30]}},$$

one can calculate the expected present value of the benefit in two ways: (i) using the D-function; and (ii) using the l-function.

The select part of A1967-70 has select period of 2 years, so $l_{[30]+10} = l_{40}$ and $D_{[30]+10} = D_{40}$.

E.P.V. of 1000 due on John's survival of 10 years:

By making use of using the D-function:

$$E.P.V. = 1000\frac{D_{[30]+10}}{D_{[30]}} = 1000\frac{D_{40}}{D_{[30]}} = 1000\frac{6986.4959}{10430.039} = 669.84 \text{ (2 d.p.)}$$

By making use of the l-function ($v = 1/1.04$ at 4% interest):

$$E.P.V. = 1000v^{10}\frac{l_{[30]+10}}{l_{[30]}} = 1000v^{10}\frac{l_{40}}{l_{[30]}} = 1000\left(\frac{1}{1.04}\right)^{10}\frac{33542.311}{33828.764} = 669.84 \text{ (2 d.p.)}$$

Identical answers!

Ignoring expenses, price=expected present value, so John should expect to pay £669.84

Now standard deviance:

$$\begin{aligned} \text{st.dev.}(z) &= 1000v^{10}\sqrt{{}_{10}p_{[30]}\ {}_{10}q_{[30]}} \\ &= 1000v^{10}\sqrt{\frac{l_{40}}{l_{[30]}}\left(1-\frac{l_{40}}{l_{[30]}}\right)} \\ &= 1000\left(\frac{1}{1.04}\right)^{10}\sqrt{\frac{33542.311}{33828.764}\left(1-\frac{33542.311}{33828.764}\right)} = 61.90 \text{ (2 d.p.)} \end{aligned}$$