# QUEEN MARY, UNIVERSITY OF LONDON

## MAS200 Actuarial Statistics

## Whole-Life Assurance: An Example Spring 2002

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**Worked example.** Suppose that a life office is about to sell the whole life assurance policy with unit death benefit to 100 individuals, age $x$, who were born on the same day.

The death benefits are to be paid immediately on the moment of death. They are to be withdrawn from an investment fund earning interest i, such that $\delta = \ln(1+i) = 0.06$.

Assume that the lives assured are subject to a constant force of mortality $\mu = 0.04$ and calculate the minimum amount to be invested at time $t = 0$, so that the probability is approximately 0.95 that sufficient funds will be on hand to withdraw the benefit payment at the death of each individual. (Assume that the future lifetimes for the lives assured are independent random variables.)

**Solution.** The minimum amount is such a value of $h$ that $P(Z \le h) = 0.95$, where $Z$ is the total present value of death benefits, $Z = \sum_{j=1}^{100} z_j$ and $z_j = v^{T_j}$.

Of course, we do not know the probability distribution of $Z$, and hence cannot find $h$ exactly. However, as 100 is a large number, we can approximate the probability distribution of

$$\frac{Z - E(Z)}{\sqrt{\operatorname{var}(Z)}}$$

by the standard normal distribution $N(0,1)$. This will allow to solve the equation $P(Z \le h) = 0.95$ for $h$.

Obviously,

$$P(Z \le h) = P\left(\frac{Z - E(Z)}{\sqrt{\operatorname{var}(Z)}} \le \frac{h - E(Z)}{\sqrt{\operatorname{var}(Z)}}\right).$$

By making use of the normal approximation for $Z$, we obtain an equation for $h$:

$$\begin{aligned} 0.95 &= P\left(\frac{Z - E(Z)}{\sqrt{\operatorname{var}(Z)}} \le \frac{h - E(Z)}{\sqrt{\operatorname{var}(Z)}}\right) \\ &\simeq P\left(N(0,1) \le \frac{h - E(Z)}{\sqrt{\operatorname{var}(Z)}}\right) = \Phi\left(\frac{h - E(Z)}{\sqrt{\operatorname{var}(Z)}}\right). \end{aligned}$$

From statistical tables we find that $\Phi(u) = 0.95$ when $u = 1.645$, therefore

$$\frac{h - E(Z)}{\sqrt{\mathrm{var}(Z)}} \simeq 1.645$$

or

$$h \simeq E(Z) + 1.645 \times \sqrt{\mathrm{var}(Z)}.$$

It remains to find the expected value and standard deviation of $Z$.

We have obtained in the lectures that $\mathrm{var}(v^{T(x)}) = \bar{A}_x^* - (\bar{A}_x)^2$ and that under the assumption of a constant force of mortality

$$\bar{A}_x \equiv E(v^{T(x)}) = \frac{\mu}{\mu + \delta}.$$

Therefore

$$E(Z) = E\left(\sum_{j=1}^{100} v^{T_j}\right) = 100\frac{\mu}{\mu+\delta} = 100 \times \frac{0.04}{0.04 + 0.06} = 40.$$

and

$$\begin{aligned} \mathrm{var}(Z) = \mathrm{var}\left(\sum_{j=1}^{100} v^{T_j}\right) &= \sum_{j=1}^{100} \mathrm{var}(v^{T_j}) \qquad \text{(as the } T_j\text{'s are independent)} \\ &= 100\left[\bar{A}_x^* - (\bar{A}_x)^2\right] \\ &= 100\left[\frac{\mu}{\mu+2\delta} - \left(\frac{\mu}{\mu+\delta}\right)^2\right] \\ &= 100 \times \left[\frac{0.04}{0.04 + 2 \times 0.06} - \left(\frac{0.04}{0.04+0.06}\right)^2\right] = 9. \end{aligned}$$

Thus finally,

$$h \simeq E(Z) + 1.645 \times \sqrt{\mathrm{var}(Z)} = 40 + \sqrt{9} \times 1.645 = 44.935.$$

Therefore one has to invest at least £44.94 to be 95% confident that sufficient funds will be on hand to pay each of the benefit payments.

If the sum assured is £10,000 then by proportion the minimum amount required is £10,000$\times$ 44.935=£44,935.