ASSIGNMENT 8

For handing in on **27 March 2002**

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the blue post-box on the second floor in the Maths building before 14:00 on Wednesday.

1. (from the 1998-1999 exam paper) Answer (a) – (c) on the basis of the A1967–70 life insurance table with 4% interest. Give your answers to 4 decimal places.

(a) A whole-life annuity-due, issued to a life age 30, pays 1 unit of money annually. Find the expected present value of the annuity. Use select values for age 30.

(b) A whole-life assurance, issued to a women age 30, has a sum assured of £72,000. The sum assured is payable at the end of the year of death. Find the expected present value of the benefit payment. Use select values for age 30.

(c) Premiums for the whole-life assurance in (b) are paid annually in advance for life. Calculate the annual premium.

2. Consider a population of insects. In this population, every insect produces offspring and dies in one year's time from birth. The number of new insects produced by one insect is subject to random variations.

Denote by s_j the number of offspring produced by insect j. Assume that the random numbers s_j are independent and have a common distribution with mean value 4.

A remote island is free from insects of any kind. Suppose that one insect from the population described above has found its way to this island. Let $Y(k)$ be the number of insects on the island k years after this event.

(a) Express $Y(k)$ through $Y(k-1)$.

(b) Express $E(Y(k)|Y(k-1))$ in terms of $Y(k-1)$.

(c) Obtain the relation between $E(Y(k))$ and $E(Y(k-1))$.

(d) On average, how many insects are on the island 10 years after the first one got there?

3. Consider a population of beetles. In this population, no beetle can survive three complete years, and for each beetle, probability that it survives its first year of life is 1/2 and the probability it survives its second year of life, given it survives the first, is 1/3. Females in this population of beetles propagate in their third year of life only, each producing on average 6 new living females.

(a) What is the probability that a newborn beetle survives to its third year of life?

(b) Write the Leslie matrix M for this population of beetles.

(c) Suppose that there are a thousand females in each age group at time $t = 0$. Find the expected numbers of females in each age group at times $t = 1$, $t = 3$ and $t = 15$.