ASSIGNMENT 7

For handing in on **20 March 2002**

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the blue post-box on the second floor in the Maths building before 14:00 on Wednesday.

This assignment is based on the material covered in Lectures 22-27. Additional reading: Neill pp. 38 –50, 73 –91 and/or Bowers et al. pp. 108 – 124 and 133 – 154.

1. On the day of his retirement at age 65 John intends to purchase an annuity that will pay him £2000 every birthday for life, the first payment being due on his 66th birthday. Ignoring any expenses, what price should he expect to pay? Assume select mortality of A1967-70 and 4% interest.

2. John is about to retire at age 65. His pension scheme provides a lump sum of £40,000 on retirement. John intends to spend this money entirely on purchasing a whole-life *immediate* (payments in arrears) annuity which provides for a monthly income. On the basis of A1967-70 ultimate values with 4% interest, estimate the monthly income; use the approximate relation $a_x^{(p)} \simeq a_x + \frac{p-1}{2p}$ and ignore expenses. Give your answer to 2 decimal places.

3. The prize in a competition can be taken in one of two ways: (i) either as the whole-life assurance with a death benefit of £2000 payable at the end of the year of death; or (ii) as £90 a year payable for life with immediate first payment. Assuming 4% interest and non-select (ultimate) mortality of A1967-70, find the range of ages for which the whole-life assurance is preferable (in the sense that it has better money value).

4(a) Consider an n-year temporary life annuity-due with $p$thly payment at rate 1 p.a., i.e. a level annuity-due, contingent on the survival of (x), payable $p$thly in advance at rate 1 p.a. for at most of n years (the maximum number of payment allowed is np). Denote by $\ddot{a}^{(p)}_{x:\overline{n}|}$ its expected present value. Show that

$$\ddot{a}^{(p)}_{x:\overline{n}|} = \frac{1}{p} \sum_{k=0}^{np-1} \frac{D_{x+\frac{k}{p}}}{D_x}$$

(b) Consider an n-year term endowment policy such that

- it provides for a benefit either on the death of (x) or on the survival of (x) to the end of the n-year term whichever event occurs first,
- the death benefit is payable at the end of the pth a year of death
- the death and survival benefits are both of 1 unit of money

Let $A^{(p)}_{x:\overline{n}|}$ be the expected present value of the benefits under this policy. Show that

$$A^{(p)}_{x:\overline{n}|} = 1 - d^{(p)}\, \ddot{a}^{(p)}_{x:\overline{n}|}.$$