ASSIGNMENT 5

For handing in on **21 February 2002**

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the blue post-box on the second floor in the Maths building before 9:45 on Thursday.

This assignment is based on the material covered in Lectures 10-14. Additional reading: Bowers at al. pp. 74 – 77 and Neil pp. 30 – 35.

1. Calculate the following probabilities on the basis of the Life Assurance Table A1967–70. Give your answers to 4 decimal places. Explain (a), (b) and (c) in words.

(a) $q_{[60]}$; **(a)** $q_{[60]+1}$; **(c)** ${}_2p_{[60]}$; **(d)** ${}_{3|1}q_{[60]}$.

2. Mr. Smith retires through ill-health at the age of 50. His future mortality is that of English Life Table No. 12 – Males, except that for the first 2 years his chance of surviving the year is three-quarters of that of the value in the table, i.e. $p_{[50]+n} = \frac{3}{4}p_{50+n}$, for $n = 0,1$ and $p_{[50]+n} = p_{50+n}$, for $n \geq 2$.

(a) Compute $p_{[50]}$ and $p_{[50]+1}$ to 5 decimal places.

(b) Extrapolate the value of $l_{[50]}$ from l_{52} using $p_{[50]+1} = \dfrac{l_{52}}{l_{[50]+1}}$ and $p_{[50]} = \dfrac{l_{[50]+1}}{l_{[50]}}$. Perform your calculations to 3 decimal places.

(c) Express $e_{[50]}$ in terms of $p_{[50]}$, $l_{[50]}$, l_{51} and e_{51}.

(d) Find the complete expectation of Mr. Smith's further life on his retirement through ill-health, i.e. calculate $\mathring{e}_{[50]}$. Use the values obtained in (a) and (b) and give your answer to 2 decimal places.

3. Mr. Smith, age 63, has just retired due to ill-health and is assumed to be subject to a constant force of mortality of 0.2931 for one year after the retirement and then will be subject to the mortality of English Life Table No. 12 – Males. Find the complete expectation of Mr. Smith's further life on his retirement through ill-health, i.e. calculate $\mathring{e}_{[63]}$. Give your answer to 2 decimal places.

4. (Exponential interpolation) Prove that if the force of mortality is constant in the age interval $[x, x+1]$, i.e. $\mu(x+t) = \mu$ for all $0 \leq t \leq 1$, then $s(x+t) = [s(x)]^{1-t}[s(x+1)]^t$ for all $0 \leq t \leq 1$.

Hint: $\dfrac{s(x+t)}{s(x)} = {}_tp_x$ and ${}_tp_x = e^{-\int_x^{x+t}\mu(u)\,du} = e^{-\int_0^t \mu(x+\tau)\,d\tau}$

5. Prove that $e_1 \geq e_0 - 1$ always. When $e_1 = e_0 - 1$?