MAS200 Actuarial Statistics

QMUL, Spring 2002

## ASSIGNMENT 4 For handing in on 14 February 2002

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the blue post-box on the second floor in the Maths building before 9:45 on Thursday.

This assignment is based on the material covered in Lectures 10-15. Additional reading: Bowers et al. pp. 51 - 57 or Neil pp. 1 - 21. Give your answers to 2 decimal places.

**1.** For a certain population of beetles, the probability for a newborn beetle to survive to the age of x years is  $\frac{1}{16}(16-x^2)$  and zero if  $x \ge 4$ . Construct the life table for this population of beetles with a radix of 1000; provide columns of  $l_x$ ,  $d_x$ ,  $p_x$ ,  $q_x$ , and  $e_x$  for ages x = 0, 1, 2, 3. (Age 0 corresponds to the newborns.) Using the constructed life table answer (a) – (c).

- (a) Estimate the proportion of beetles surviving their first year of life.
- (b) For a group of 500 newborn beetles, estimate the number of beetles dying in their second year of life.
- (c) Estimate the average time-until-death for the one-year-old beetles.

**2.** Assume that the survival function is the same as in Question 1. Obtain an analytic expression for  $_tp_1$ , the probability for a two-year-old beetle to survive the following *t* years. Using the relation  $\mathring{e}_x = \int_t p_x dt$ , obtained in Lecture 13, find the complete expectation of life at the age of 1. Compare your answer with the life table value obtained in 1(c).

**3.** Answer (a) - (e) on the basis of the English Life Table No. 12 - Males.

- (a) What is the probability that (40) survives the following 10 years?
- (b) What is the probability that (40) fails to attain the age of 70?
- (c) What is the probability that (40) survives to the age of 65 and dies before attaining the age of 70?
- (d) What is the probability for a newborn to die between the ages of 65 and 70?

**4.** The inhabitants of a remote island are subject to mortality of the English Life Table No. 12 – Males. For a random survivorship group of 1000 newborns find:-

- (a) the expected number of survivors to the age of 60 in the group;
- (b) the expected number of those in the group who will die aged 60 or 61 last birthday.

5. (Optional, no marks here) According to the ELT-12-Males, the complete expectation of life at birth,  $e_0$ , is 68.09 years, i.e. the expected lifetime of a newborn is 68.09 years. A student argues that the complete expectation of life at age 1 is simply one year less than the complete expectation of life at birth, as at age 1 the newborn has already lived one year. Can you explain where the student got it wrong?