MAS200 Actuarial Statistics

## ASSIGNMENT 3 For handing in on 7 February 2002

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the blue post-box on the second floor in the Maths building before 9:45 on Thursday.

This assignment is based on the material covered in Lectures 10-12. Additional reading: Bowers et al. pp. 51 - 57 or Neill pp. 1 - 21.

**1.** In a certain valley in Georgia, the probability for a newborn baby to survive to age x is  $1 - \frac{x^2}{100^2}$  if  $0 \le x \le 100$ , and zero if x > 100 (i.e. nobody in the valley lives more than 100 years.)

- (a) Write down the survival function for this population
- (b) Express the events in (i)-(iv) through the random variable X (the future lifetime at birth) and find their probabilities. Give your answers to two decimal places.
  - (i) newborn baby survives to the age of 50;
  - (ii) person aged 20 survives to the age of 50;
  - (iii) person aged 20 dies between the ages of 50 and 70;
  - (iv) newborn baby dies between the ages of 50 and 70.
- (c) Find the force of mortality at age *x*, for all 0 < x < 100. Calculate to four decimal places the force of mortality at the age of 95.

For a group of 1000 Georgians who attained the age of 95, estimate expected number of those who will die within one week after the 95th birthday. Round your answer to the nearest integer.

**2.** The inhabitants of a remote island are subject to a constant force of mortality  $\mu = 0.21$  in the age interval from 70 to 80 years. For this population, find an analytic expression for  $_t p_x$  in the range  $70 \le x < x + t \le 80$ . For a person aged 72, calculate to two decimal places

- (a) the probability of survival to the age of 76;
- (b) the probability of death within the following 3 years;
- (c) the probability of death between the ages of 76 and 79.

**3.** Assume that the survival function is the same as in Question 1.

- (a) What is the range of values of the random variable T(50), the time-until-death at age 50? Obtain  $f_{T(50)}(t)$ , the probability density function of T(50). Verify that the obtained expression satisfies  $\int_0^{+\infty} f_{T(50)}(t) dt = 1$ .
- (b) Calculate to two decimal places the complete expectation of future life at the age of 50.