

Q1/6

M3

1 mark for each
one of each
so that

1*. The annual premium is $\mathcal{L}P$, where $P\ddot{a}_{[27]:\overline{23}|} = 50,000A_{[27]:\overline{23}|}$.

We use

$$A_{[27]:\overline{23}|} = 1 - d\ddot{a}_{[27]:\overline{23}|},$$

$$\text{and } \ddot{a}_{[27]:\overline{23}|} = \frac{N_{[27]} - N_{50}}{D_{[27]}},$$

$$\begin{aligned} P &= \frac{50,000 \times A_{[27]:\overline{23}|}}{\ddot{a}_{[27]:\overline{23}|}} \\ &= 50,000 \left(\frac{D_{[27]}}{N_{[27]} - N_{50}} - d \right) \\ &= 50,000 \left(\frac{11755.233}{253646.27 - 73567.136} - \frac{4}{104} \right) = 1340.830654 \end{aligned}$$

A1

Hence the annual premium is $\mathcal{L}1,340.83$

M1

If he is only able to pay a premium of $\mathcal{L}500$ per annum, then he can purchase $\mathcal{L}C$ of 23-year endowment assurance, where $500\ddot{a}_{[27]:\overline{23}|} = C \times A_{[27]:\overline{23}|}$. Therefore

$$C = \frac{500 \times \ddot{a}_{[27]:\overline{23}|}}{A_{[27]:\overline{23}|}} = \frac{500}{\left(\frac{D_{[27]}}{N_{[27]} - N_{50}} - d \right)} = \frac{500}{\left(\frac{11755.233}{253646.27 - 73567.136} - \frac{4}{104} \right)} = 18,645.16$$

A1

$\mathcal{L}500$ per annum paid in advance will buy $\mathcal{L}18,645.16$ of 23-year endowment assurance for a life aged 27.

Alternatively: a premium of $\mathcal{L}1340.830654$ buys $\mathcal{L}50,000$ of endowment assurance, so a premium of $\mathcal{L}500$ buys $\mathcal{L}(500/1340.830654) \times 50,000 = \mathcal{L}18,645.16$ of endowment assurance.

Q2/6

M2

2*. The cost C (in pounds) of the endowment assurance in question is

$$C = 25000A_{[40]:\overline{25}|}^{\frac{1}{25}} + 70000A_{[40]:\overline{25}|}^1,$$

where $A_{[40]:\overline{25}|}^{\frac{1}{25}}$ and $A_{[40]:\overline{25}|}^1$ are, respectively, the expected present values of one unit of the survival benefit and one unit of the death benefit.

To work out the required annual premium P (in pounds), equate the cost of the endowment assurance to the cost (the expected present value) of the annuity of premiums. The latter is $P\ddot{a}_{[40]:\overline{25}|}$, so that

$$P = \frac{25000A_{[40]:\overline{25}|}^{\frac{1}{25}} + 70000A_{[40]:\overline{25}|}^1}{\ddot{a}_{[40]:\overline{25}|}}.$$

Recalling that

$$A_{[40]:\overline{25}|}^{\frac{1}{25}} = \frac{D_{[40]+25}}{D_{[40]}}$$

$$A_{[40]:\overline{25}|}^1 = 1 - \frac{D_{[40]+25}}{D_{[40]}} - d \frac{N_{[40]} - N_{[40]+25}}{D_{[40]}}$$

$$\ddot{a}_{[40]:\overline{25}|} = \frac{N_{[40]} - N_{[40]+25}}{D_{[40]}}$$

we obtain the annual premium in terms of the monetary functions D and N :

$$\begin{aligned} P &= 25000 \frac{D_{[40]+25}}{N_{[40]} - N_{[40]+25}} + 70000 \left(\frac{D_{[40]}}{N_{[40]} - N_{[40]+25}} - \frac{D_{[40]+25}}{N_{[40]} - N_{[40]+25}} - d \right) \\ &= 70000 \left(\frac{D_{[40]}}{N_{[40]} - N_{[40]+25}} - d \right) - 45000 \frac{D_{[40]+25}}{N_{[40]} - N_{[40]+25}}. \end{aligned}$$

From the A1967-70 table,

$$\begin{aligned} D_{[40]+25} &= D_{65} = 2144.1713 \\ N_{[40]+25} &= N_{65} = 23021.434 \\ D_{[40]} &= 6981.5977 \\ N_{[40]} &= 131995.19, \end{aligned}$$

and

$$\begin{aligned} P &= 70000 \left(\frac{6981.5977}{131995.19 - 23021.434} - \frac{4}{104} \right) - 45000 \frac{2144.1713}{131995.19 - 23021.434} \\ &= 906.95 \quad (\text{to 2 d.p.}) \end{aligned}$$

A1

Hence the annual premium is £906.95.

Q317

M1

3*. (a) P.V. of the benefit payment

$$Z = \begin{cases} 6000v^{18}, & \text{if } T(0) > 18 \\ 0, & \text{otherwise} \end{cases}$$

or equivalent
accept $x > 18$

where $T(0)$ is the exact future lifetime for a newborn and $v = \frac{1}{1+i} = \frac{1}{1.1}$.

Alternatively, $Z = 6000v^{18} \times \text{Bernoulli}(p)$ where $p = P(T(0) > 18) = {}_{18}p_0$.

A1

$$\begin{aligned} \text{E.P.V.} = E(Z) &= 6000v^{18}P(T(0) > 18) \\ &= 6000v^{18} {}_{18}p_0 = 6000v^{18} \frac{l_{18}}{l_0} = 6000 \left(\frac{1}{1.1} \right)^{18} \frac{96514}{100000} = \underline{1041.53} \end{aligned}$$

$\text{var}(Z) = (6000v^{18})^2 ({}_{18}p_0)({}_{18}q_0)$. Hence

$$\begin{aligned} \text{st. dev.}(Z) &= 6000v^{18} \sqrt{({}_{18}p_0)({}_{18}q_0)} = 6000v^{18} \sqrt{\frac{l_{18}}{l_0} \left(1 - \frac{l_{18}}{l_0} \right)} \\ &= 6000 \left(\frac{1}{1.1} \right)^{18} \sqrt{\frac{96514}{100000} \left(1 - \frac{96514}{100000} \right)} = \underline{197.94} \end{aligned}$$

A1

(b) Denote by Y the total present value of the benefit payments, $Y = \sum_{j=1}^{100} Z_j$, where Z_j is the present value of the benefit payment for boy j .

Then

$$\begin{aligned} E(Y) &= 100 \times E(Z) = 100 \times 1041.53 = 104153 \\ \text{var}(Y) &= 100 \text{var}(Z) \quad [\text{as } Z_j \text{ are independent and have the same distribution}] \\ \text{st. dev.}(Y) &= 10 \times \text{st. dev.}(Z) = 10 \times 197.94 = 1979.4 \end{aligned}$$

The minimum amount is such value h that $P(Y \leq h) = 0.95$.

$$P(Y \leq h) = P\left(\frac{Y - E(Y)}{\sqrt{\text{var}(Y)}} \leq \frac{h - E(Y)}{\sqrt{\text{var}(Y)}} \right) = 0.95$$

M2 for explanations

Normal approximation: $\frac{Y-E(Y)}{\sqrt{\text{var}(Y)}} \sim N(0,1)$. As $P(N(0,1) \leq 1.645) = 0.95$ we obtain an equation for h :

$$\frac{h - E(Y)}{\sqrt{\text{var}(Y)}} = 1.645$$

Hence,

M1 $h = E(Y) + 1.645 \times \text{st. dev.}(Y) = 104153 + 1.645 \times 1979.4 = \underline{107409.1}$ (1 d.p.)

A1 The minimum amount to be invested is $\underline{\pounds 107409.1}$

Q4/6

4*. (a)

M1 $Y = \begin{cases} a_{\overline{K(x)|}} & \text{if } 0 \leq K(x) \leq n-1 \\ a_{\overline{n|}} & \text{if } K(x) \geq n \end{cases}$

also accept $Y = \begin{cases} a_{\overline{K(x)|}} & \text{if } 0 \leq K(x) \leq n \\ a_{\overline{n|}} & \text{if } K(x) \geq n+1 \end{cases}$

also accept if write $Y=0$ if $K(x)=0$.
also accept if write

(b)

M1 $Z = \begin{cases} v^{K(x)+1} & \text{if } 0 \leq K(x) < n+1 \\ v^{n+1} & \text{if } K(x) \geq n+1 \end{cases}$

Y can also be written as

$$Y = \begin{cases} a_{\overline{K(x)|}} & \text{if } 0 \leq K(x) \leq n \\ a_{\overline{n|}} & \text{if } K(x) \geq n+1 \end{cases}$$

making the comparison with Z easier.

M1 If $0 \leq K(x) \leq n$ then $Z = v^{K(x)+1}$ and $Y = \frac{v(1-v^{K(x)})}{(1-v)} = \frac{v}{1-v} + \frac{v^{K(x)+1}}{1-v}$, so that $Y = \frac{v}{1-v} - \frac{Z}{1-v}$.

If $K(x) \geq n+1$ then $Z = v^{n+1}$ and $Y = \frac{v(1-v^n)}{(1-v)} = \frac{v}{1-v} + \frac{v^{n+1}}{1-v}$, so that $Y = \frac{v}{1-v} - \frac{Z}{1-v}$.

(c)

M1 $E[Y] = \frac{v}{1-v} - \frac{E[Z]}{1-v} = \frac{v}{1-v} - \frac{A_{\overline{x:n+1}|}}{1-v}$

M2 $\text{var}(Y) = \frac{1}{(1-v)^2} \text{var}(Z) = \frac{1}{(1-v)^2} (E[Z^2] - (E[Z])^2) = \frac{1}{(1-v)^2} (A_{\overline{x:n+1}|}^* - (A_{\overline{x:n+1}|})^2)$

where (as in lectures) * indicates that v is replaced by $v^* = v^2$ so that the interest rate is $i^* = (1+i)^2 - 1$.