MAS224, Actuarial Mathematics: Problem Sheet 7

*Post your solutions to the starred questions in the **orange box** on the **second floor** of the Maths building by **12 noon on Monday, 17th March 2008**. Do not forget to staple all pages together and write your name and student number at the top of the front sheet.*

In problems 1 and 2 assume a 4% interest rate and the mortality given by table A1967-70 select values. Give your numerical answers to the nearest penny.

1*. A life aged 27 takes out a 23-year endowment assurance policy which pays $£50,000$ benefit on death or survival for 23 years. Premiums are made annually in advance (contingent on survival), there being a maximum of 23 possible payments. Calculate the premium required.

If he is only able to pay a premium of $£500$ per annum, how much 23-year endowment assurance will this purchase?

2*. A life aged 40 takes out a 25-year endowment assurance which pays $£25,000$ benefit on survival for 25 years and $£70,000$ benefit on death if it occurs before the 65th birthday. The death benefit is paid out at the end of the year of death. Premiums are paid annually in advance. Find the annual premium.

3*. A population is subject to mortality described by English Life Table No. 12 – Males.

(a) Consider a pure endowment policy for a boy, effected at his birth, which provides £6,000 for the boy on his 18th birthday. Write down the present value of the benefit payment under this policy, regarding it as a random variable. Calculate its mean value and standard deviation (the square root of variance) to 2 decimal places.
Assume the effective interest rate of 10% per annum.

(b) A life company withdraws benefit payments from an investment fund earning interest of 10% per annum effectively. Suppose that the company has just sold a block of 100 pure endowment policies identical to that one in (a). Using the results obtained in (a), calculate the minimum amount to be invested by the company at the present time so that the probability is approximately 0.95 that sufficient funds will be on hand to withdraw the benefit payment for each boy; use a normal approximation for the total present value of the benefit payments and show your working.
(If N has the standard normal distribution then $P(N < 1.645) = 0.95$.)

4*. (a) Consider an n-year immediate life annuity of 1 unit per year paid annually. The annuitant is aged x. Let Y be the present value of the annuity. Write Y in terms of $K(x)$.

(b) Consider an (n+1)-year endowment assurance taken out by the same life aged x with death and survival benefits of 1 unit, with the death benefit payable at the end of the year of death. Let Z be the present value of the endowment assurance. Write Z in terms of $K(x)$. Show that the relationship between Z and Y from part (a) is just $Y = \frac{v}{1-v} - \frac{Z}{1-v}$

(c) Use the result from lectures for the mean and variance of the present value of an endowment assurance, with the relation between Y and Z in part (b), to find $E[Y]$ and $Var(Y)$.