MAS224, Actuarial Mathematics: Problem Sheet 6

Post your solutions to the starred questions in the ***orange box*** *on the* ***second floor*** *of the Maths building by* ***12 noon on Monday, 10th March 2008****. Do not forget to staple all pages together and write your name and student number at the top of the front sheet.*

In problems 1 and 2 assume a 4% interest rate and the mortality given by table A1967-70 select values. Give your numerical answers to the nearest penny.

1*. (a) Find the cost (the expected present value) of a whole-life assurance with death benefit of $\pounds 50,000$ payable at the end of the year of death taken out by a life age 28.

(b) Calculate the premium required to purchase the policy in part (a) if the premium is to be paid annually in advance during the lifetime of the life assured.

2*. John Doe receives a lump sum of £50,000 on retirement at age 60. He uses this to purchase a whole-life immediate annuity (the first payment is due on his 61st birthday subject to survival). Find the annual payments he will receive.

3*. Consider a whole-life annuity-due of 1 unit of money per year to a life who is age x now. Suppose that the first payment is deferred for m years and is contingent of the survival of (x) to age $x+m$. Let Z be the present value of the annuity. Then Z is a discrete random variable taking values 0 and $v^m \ddot{a}_{\overline{k+1}|}$, $k = 0, 1, 2, \ldots$.

(a) Describe the event $Z = v^m \ddot{a}_{\overline{k+1}|}$ in terms of $K(x)$, the curtate further lifetime at age x. Express the probabilities $P(Z = v^m \ddot{a}_{\overline{k+1}|})$, $k = 0, 1, 2, \ldots$, in terms of the life-table function ${}_tp_y$.

(b) Using the results derived in part (a), obtain $E(Z) = \frac{N_{x+m}}{D_x}$, where $D_x = v^x l_x$ and $N_x = \sum_{k=0}^{\infty} D_{x+k}$. [Hint: follow the derivation of $\ddot{a}_x = \frac{N_x}{D_x}$ in Lecture 21.]

4*. Consider a whole-life assurance policy to a life age x with sum assured (death benefit) of 1 unit of money. Let Z_1 be the present value of the benefit payment if it is due at the instant of death and Z_2 be the present value of the benefit payment if it is due at the end of the year of death, so that $\bar{A}_x = E(Z_1)$ and $A_x = E(Z_2)$.

(a) Express Z_1 in terms of $T(x)$, the exact further lifetime at age x. State the p.d.f. of $T(x)$ in terms of the survival function $s(x)$. Show that

$$\bar{A}_x = -\frac{1}{s(x)} \sum_{k=0}^{\infty} \int_0^1 v^{t+k} \frac{d}{dt} s(x+k+t)\, dt.$$

(b) Use linear interpolation on the survival function to obtain the approximation

$$\bar{A}_x \approx \left(\int_0^1 v^t dt \right) \sum_{k=0}^{\infty} v^k ({}_kp_x - {}_{k+1}p_x).$$

Hence show that $\bar{A}_x \approx \frac{i}{\delta} A_x$ where δ is the force of interest and i is the annual rate of interest.