MAS224, Actuarial Mathematics: Problem Sheet 5

*Post your solutions to the starred questions in the **orange box** on the **second floor** of the Maths building by **12 noon on Monday, 25th February 2008**. Do not forget to staple all pages together and write your name and student number at the top of the front sheet.*

1*. Let x be a non-negative integer and $0 < t < 1$. Assume a uniform distribution of deaths within each year of life (so linear interpolation on $s(x)$ is appropriate).

Show that ${}_tp_x = 1 - t + tp_x$ and ${}_{1-t}p_{x+t} = \frac{p_x}{1-t(q_x)}$.

2*. Assume the mortality of ELT12. Use linear interpolation on $s(x)$, and hence l_x, to calculate the following to 4 significant digits:

(a) Find the probability that a man who has just reached his 18th birthday will die within the next 3 months.

(b) Find the probability that a man who has just reached his 50th birthday will survive the next 6 months.

(c) Find the probability that a man who has just reached his 60th birthday will die within the month preceding his 65th birthday.

(d) A man retires exactly 6 months after his 60th birthday. Find his (non-curtate) expected further lifetime.

(e) If there are 1000 newborns, find the expected number who die within 9 months (either way) of their 21st birthday.

3*. A remote region has a stable population with the expected number aged x in the population, n_x, given by $n_x = Al_x$ where l_x is the value in table ELT12 and A is a suitable scaling factor.

If the expected population size is $5,000$, find:

(i) the scaling factor A;

(ii) the expected number of births each year;

(iii) the expected number in the population who are aged under 60.

4*. Use the table A1967-70 of the select mortality function l to find the following:

(a) ${}_{34}p_{[26]}$ (b) ${}_4q_{[65]+1}$ (c) ${}_{9|1}q_{[65]}$.

(d) 100 men each take out a pension plan on their 23rd birthday. Retirement is at their 60th birthday. Find the expected number who die within a year of retirement.

(e) A man takes out a life assurance policy on his 30th birthday for which he is to pay by monthly installments starting immediately. Payments continue during his lifetime. Find the probability that he only makes the first four payments (assume a uniform distribution of death between integer ages).

5*. Calculate a mini version of the table with entries the select mortality functions $l_{[x]}$, $l_{[x]+1}$ and l_{x+2} for $x = 63, 64, 65$ and based on ELT12. You should take $p_{[x]} = 0.7 \times p_x$ and $p_{[x]+1} = 0.9 \times p_{x+1}$.

Calculate $\mathring{e}_{[63]}$, $\mathring{e}_{[63]+1}$ and $\mathring{e}_{[64]}$.