MAS224, Actuarial Mathematics: Problem Sheet 4

*Post your solutions to the starred questions in the **orange box** on the **second floor** of the Maths building by **12 noon on Monday, 11th February 2008**. Do not forget to staple all pages together and write your name and student number at the top of the front sheet.*

Give answers to 4 decimal places.

1*. Suppose that beetles in a particular population are certain to die before they reach age 5. The chance of surviving one year is known for each age, the values being $p_0 = 0.9,\ p_1 = 0.5,\ p_2 = 0.2,\ p_3 = 0.1$ and $p_4 = 0$.

Construct their life table giving $p_x,\ q_x,\ l_x,\ d_x$ and $\mathring{\mathsf{e}}_x$ using a radix of $l_0 = 1,000$.

Find ${}_3p_1$, ${}_2q_2$ and ${}_{2|1}q_1$.

Let $K(x)$ be the curtate further lifetime of a beetle aged x. Find $P(K(1) = k)$ for each value of k for which the probability is non-zero.

2*. Use the life table ELT12 to calculate the following:

(a) The probability for a man age 60 to die during the next five years.

(b) The probability for a man age 40 to die by age 65.

(c) The probability for a man age 20 to survive to age 50 but die before age 60.

(d) The expected number of deaths of men within 1 year of retirement at age 65 out of 100,000 newborns.

(e) The expected number of men who die by age 30 out of 1,000 newborns.

(f) The expected number of men who will survive to age 65 out of 1,000 who are aged 50 now.

3. Consider a population where individuals live at most 6 years and where the survival function $s(x) = 1 - \frac{x^2}{36}$ for $0 \leq x \leq 6$.

(a) Construct the life table giving $p_x,\ q_x,\ l_x,\ d_x$ and e_x using a radix of $l_0 = 3,600$.

(b) Find the force of mortality (instantaneous death rate) $\mu(x)$ for $0 < x < 6$.

(c) Find $\mathring{\mathsf{e}}_x$ for $0 \leq x < 6$.

4*. Use the expression derived in lectures for e_x to show that $\mathsf{e}_x \leq 1 + \mathsf{e}_{x+1}$. When does $\mathsf{e}_x = 1 + \mathsf{e}_{x+1}$?

5*. In a population no individual lives beyond 50 years and the instantaneous rate of death (force of mortality) is given by $\mu(x) = \frac{2}{50-x}$ for $0 < x < 50$. Find $s(x)$ for $0 \leq x \leq 50$. Find the probability density function for the further life time $T(x)$ at age x. Calculate $\mathring{\mathsf{e}}_{20}$, the complete expectation of life at age x.