LECTURE 18

Life Assurances (continued)

n-year term life assurance with death benefit payable on the moment of death

Assume unit death benefit payable immediately on death of (x) if the death occurs within n years (i.e. within the term of policy). Then the present value of this benefit is

$$z = \begin{cases} v^{T(x)}, & \text{if } T(x) \leq n \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

The expectation of z (E.P.V.) is denoted by the symbol $\bar{A}^1_{x:\overline{n}|}$ and

$$\begin{aligned} \bar{A}^1_{x:\overline{n}|} = E(z) &= \int_0^\infty \left\{ \begin{array}{ll} v^t, & \text{if } t \leq n \\ 0, & \text{otherwise} \end{array} \right\} f_{T(x)}(t)\,dt \\ &= \int_0^n v^t\,{}_tp_x \mu(x+t)\,dt. \end{aligned}$$

$$\text{var}(z) = E(z^2) - [E(z)]^2 = \bar{A}^1_{x:\overline{n}|@i^*} - \left(\bar{A}^1_{x:\overline{n}|}\right)^2.$$

where $i^* = 2i + i^2$ (so that $v^* = v^2$.)

Pure Endowment

Assume unit benefit payable immediately on the survival of (x) to age $x+n$. The present value of benefit is

$$z = \begin{cases} 0, & \text{if } T(x) \leq n \\ v^n, & \text{if } T(x) > n \end{cases} \tag{2}$$

z is a discrete-type random variable (actually, $z = v^n \times \text{Bernoulli}({}_np_x)$). Its expectation (E.P.V.) is denoted by the symbol $A_{x:\overset{1}{\overline{n}|}}$ and

$$A_{x:\overset{1}{\overline{n}|}} = E(z) = v^n P(T(x) > n) = v^n\,{}_np_x$$

$$\text{var}(z) = (v^2)^n\,{}_np_x\,{}_nq_x = (v^*)^n\,{}_np_x\,{}_nq_x.$$

n-year term endowment policy with death benefit payable on the moment of death

This a combination of an n-year term life assurance policy and a pure endowment. We shall consider the case when the death and survival benefits are equal.

Suppose the benefits are of 1 unit of money each. Then total present value of the benefits under this policy is $z = z_1 + z_2$ where z_1 is the P.V. of the death benefit (given by Eq. (1)) and z_2 is the P.V. of the survival benefit (given by Eq. (2):

$$z = \begin{cases} v^{T(x)}, & \text{if } T(x) \le n \\ v^n, & \text{if } T(x) > n \end{cases}$$

Its expected present value is denoted by the symbol $\bar{A}_{x:\overline{n}|}$ and

$$\bar{A}_{x:\overline{n}|} = E(z) = \bar{A}^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\;\;\,1}. \tag{3}$$

Eq. (3) follows from $E(z_1 + z_2) = E(z_1) + E(z_2)$ and the previously obtained expressions for $E(z_1)$ and $E(z_2)$.

Commutation Functions

Commutation functions are used for calculating E.P.V. of life assurances on the basis of life table values.

Commutation functions involve both mortality and interest.

We shall consider only two commutation functions D_x and N_x. The are defined as follows

$$D_x = v^x l_x \quad N_x = \sum_{k=0}^{\infty} D_{x+k}.$$

Values of these commutation functions for integer x are provided in the life assurance tables.

Example.

John, age 30, is taking out a pure endowment policy with a term of 10 years. Under this policy John receives 1 unit of money on his survival to age 40. Assume 4% interest and calculate the expectation and variance of the present value of John's benefit on the basis of A1967-70. Use select values for age 30.

Solution.

The E.P.V. of unit benefit to be paid to (x) if he survives from age x to age $x+n$ is

$$A_{x:\overline{n}|}^{\;\;\,1} = v^n\, {}_np_x = v^n \frac{l_{x+n}}{l_x} = \frac{D_{x+n}}{D_x}.$$

where $D_x = v^x l_x$.

Have to find $A_{[30]:\overline{10}|}^{\;\;1}$ on the basis of A1967-70. This table has a select period of 2 years, hence $[30]+10 = 30+10 = 40$.

We can calculate $A_{[30]:\overline{10}|}^{\;\;1}$ in two ways: (i) using the D function; and (ii) using the l function.

Using D_x:

$$A_{[30]:\overline{10}|}^{\;\;1} = \frac{D_{[30]+10}}{D_{[30]}} = \frac{D_{40}}{D_{[30]}}$$
$$= = \frac{6986.4959}{10430.039} = 0.6698,$$

Using l_x:
Need to calculate v first. The interest rate is 4%, hence $v = 1/(1+i) = 1/(1+0.04)$. Now

$$A_{[30]:\overline{10}|}^{\;\;1} = v^{10}\,{}_{10}p_{[30]} = v^{10}\frac{l_{[30]+10}}{l_{[30]}} = v^{10}\frac{l_{40}}{l_{[30]}}$$
$$= \left(\frac{1}{1.04}\right)^{10}\frac{33542.311}{33828.764} = 0.6698$$

the same answer as in the above.

Now variance:

$$\text{var}(P.V.) = (v^2)^{10}\,{}_{10}p_{[30]}\,{}_{10}q_{[30]} = v^{20}\,{}_{10}p_{[30]}(1-{}_{10}p_{[30]})$$
$$= v^{20}\frac{l_{40}}{l_{[30]}}(1-\frac{l_{40}}{l_{[30]}})$$
$$= \left(\frac{1}{1.04}\right)^{20}\frac{33542.311}{33828.764}\left(1-\frac{33542.311}{33828.764}\right) = 0.0038$$