LECTURE 16

Life Assurances

Glossary

Life assurance policy:

this is a contract which pays a specified sum (the *sum assured*, or the *death benefit*) on the death of a specified person (the *life assured*).

Whole life assurance:

this is a life assurance contract which pays the sum assured on the death of the life assured at any future time.

Temporary life assurance:

this is a life assurance contract which pays the sum assured on the death of the life assured if the death occurs within a specified period.

n-year term life assurance:

this is a temporary life assurance policy which pays the sum assured on the death of the life assured within n years from the start of the contract.

Pure endowment policy:

this is a contract which pays a benefit on the survival of the life assured until a certain date.

n year term endowment policy:

this is a policy which combines an n-year term life assurance with a pure endowment policy, i.e. it provides for a benefit either on the death or on the survival (of the life assured) to the end of the n-year term whichever event occurs first.

The benefits may be level (constant) or they may decrease or increase in the way specified in the contract.

With profit policies:

the benefits may be increased by additions called bonuses.

Without profit (non-profit) policies:

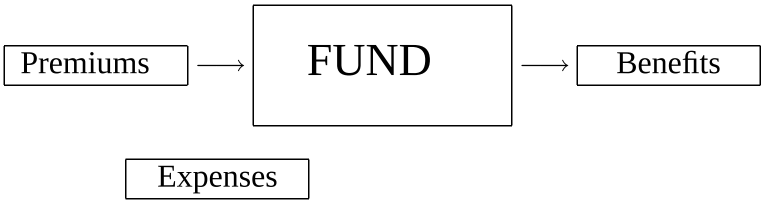
the benefits are completely specified in money terms in the contract.

Premium(s):

one-off payment or payment in regular installments made to the insurance company (the life office) in return for payment of the benefit.

In this course, we consider non-profit policies with level benefits only.

Normally, the life office invests collected premiums into a fund. This fund earns interest and is used to pay out benefits.



Premiums normally include charges. The charges are used to cover the life company's expenses. Premiums are worked out by applying the equation of value: when invested into the fund they should generate a return which then will be used to pay out the benefit and to cover the company's expenses.

At this stage, we assume no expenses. Under this assumption we should equate the present values of benefit and premiums. However, these present values are random variables, as they depend on the survival of the life assured: the benefit is paid on the death of the life assured and the premiums are normally paid whilst the life assured is alive.

As the exact time-until-death for the life assured is unknown, the premiums are worked out by equating the expected present values of the benefit and premiums:

$$\text{E.P.V. of benefit(s)} \quad = \quad \text{E.P.V. of premiums}$$

where E.P.V. stands for the expected present value.

Have to learn:

- how to calculate the E.P.V. of benefit(s);
- how to calculate the E.P.V. of premiums.

Payment of premiums can be regarded as a life annuity. We shall calculate the corresponding E.P.V.'s later on in the course.

This week and the next one we shall be calculating the E.P.V. of life assurance benefits.

Three modes of payment of the death benefit to be considered:

- death benefit payable on the moment of death (convenient theoretical abstraction)
- death benefit payable at the end of the year of death
- death benefit payable at the end of the month (quarter, week, etc.) of death.

In this lecture we calculate the mean and variance of the present value of the death benefit payable immediately on death.

Whole life assurance: *E.P.V. of the death benefit payable at the moment of death*

Assume the company's fund earns interest at rate of i per annum effectively. Hence the present value of 1 due in t years is v^t, where $v = \frac{1}{1+i}$.

Now consider a life aged x who is taking out a whole life assurance with the sum assured of one unit of money. Assume that the sum assured is paid *immediately* on the death of (x). Then its present value is

$$z = v^{T(x)},$$

where $T(x)$ is the time-until-death for (x).

$T(x)$ is a random variable with p.d.f. given by $f_{T(x)}(t) = {}_t p_x \mu(x+t)$.

Therefore

$$\begin{aligned} E(z) &= \int_0^\infty v^t f_{T(x)}(t)\,dt \\ &= \int_0^\infty v^t\, {}_t p_x \mu(x+t)\,dt \end{aligned}$$

Notation: $E(z)$ is denoted by $\bar{A}_x$,

$$\bar{A}_x \stackrel{\text{def.}}{=} E(z) = \int_0^\infty v^t\, {}_t p_x \mu(x+t)\,dt,$$

where A stands for "assurance", the bar indicates that the sum assured is paid immediately on the death of (x).

Thus, by definition, $\bar{A}_x$ is the expected present value of unit death benefit payable immediately on death of (x). If the death benefit is of S units of money, then, by proportion, its E.P.V. is

$$E(Sz) = S\bar{A}_x.$$

Whole life assurance: *Variance of theP.V. of the death benefit payable immediately on death*

Consider unit death benefit payable immediately on the death of (x). Its present value $v^{T(x)}$ is a random variable with mean $\bar{A}_x$. The variance of $v^{T(x)}$ is given by

$$\begin{aligned} \text{var}(z) &= E(v^{2T}) - [E(v^T)]^2 \\ &= \int_0^\infty v^{2t}\, {}_t p_x \mu(x+t)\,dt - [\bar{A}_x]^2 \\ &= \int_0^\infty (v^*)^t\, {}_t p_x \mu(x+t)\,dt - [\bar{A}_x]^2, \qquad \text{where } v^* = v^2 \\ &= \left[\bar{A}_x^* - (\bar{A}_x)^2\right], \end{aligned}$$

where the rate of interest i^* is such that $(v^*) = v^2$, i.e.

$$i^* = 2i + i^2.$$

If the death benefit is of S units of money, then by proportion, the variance of its present value is

$$E([Sv^T]^2) - [E(Sv^T)]^2 = S^2\left[\bar{A}_x^* - (\bar{A}_x)^2\right]$$

SUMMARY.

Whole life assurance:	present value (P.V.) of the death benefit is a random variable; the expected present value, E.P.V., of a death benefit of 1 payable immediately on the death of the life assured is denoted by $\bar{A}_x$ and $$\bar{A}_x = \int_0^\infty v^t\, {}_tp_x \mu(x+t)\, dt;$$ its variance is $\bar{A}_x^* - (\bar{A}_x)^2$, where the star indicates the interest rate $i^* = 2i + i^2$, $v^* = v^2$.