LECTURE 12

Worked Examples; Laws of mortality

Worked Example 1. ELT-12-Males

Calculate the following probabilities on the basis of the English Life Table No. 12 – Males:

(a) Probability that a newborn survives to age 18.

(b) Probability that a life aged 25 survives to age 60.

(c) Probability that a life aged 25 dies within next 35 years of life.

(d) Probability that a life aged 25 dies aged between 60 and 70.

Solution.

First express these probabilities in terms of life-table functions and then use relations (1) – (2) obtained in Lecture 11.

X is the time-until-death for a newborn. $T(x)$ is the future lifetime of (x), i.e. a person aged x, and $T(x) = X - x$ given $X > x$. Therefore:-

(a) Need to find $P(T(0)>18)$ or, equivalently, $P(X>18)$.

$$P(T(0)>18)=P(X>18)={}_{18}p_0=\frac{l_{18}}{l_0}=\frac{96514}{100000}=0.96514.$$

(b) Need to find $P(T(25)>35)$ or, equivalently, $P(X>25+35|X>25)$.

$$P(T(25)>35)=P(X>60|X>25)={}_{35}p_{25}=\frac{l_{60}}{l_{25}}=\frac{78924}{95753}=0.8242.$$

(c) Need to find $P(T(25)\le 35)$ or, equivalently, $P(25<X\le 25+35|X>25)$.

$$P(T(25)\le 35)=1-P(T(25)>35)=1-{}_{35}p_{25}=1-\frac{l_{60}}{l_{25}}=1-\frac{78924}{95753}=0.1758.$$

or

$$P(T(25)\le 35)=P(25<X\le 60|X>25)={}_{35}q_{25}=\frac{l_{25}-l_{60}}{l_{25}}=\frac{95753-78924}{95753}=0.1758.$$

(d) Need to find $P(60-25<T(25)\le 70-25)$ or, equivalently, $P(60<X\le 70|X>25)$.

$$P(35<T(25)\le 45)={}_{35|10}q_{25}=P(60<X\le 70|X>25)=\frac{l_{60}-l_{70}}{l_{25}}=\frac{78924-54806}{95753}=0.25188.$$

Worked Example 2. Random Survivorship Group

A population is subject to mortality described by the ELT-12-Males. A group of 9000 newborns were born in a specific year. Estimate the number of survivors to age 60 from the group.

Solution.

We estimate this number by the expected number of survivors to age 60, which is given by

$$9000\times P(X>60)=9000\times {}_{60}p_0=9000\frac{l_{60}}{l_0}=\frac{9000}{100000}l_{60}=\frac{9000}{10000}78924=7100.$$

The Curve of Deaths

By the definition of ${}_td_x$, ${}_ud_x$ is the expected number of deaths in the age interval between x and $x+u$ in a group of l_0 newborns.

$$\begin{aligned} {}_ud_x = l_x - l_{x+u} &= l_0 s(x) - l_0 s(x+u) \\ &\simeq -l_0\left[\frac{d}{dx}s(x)\right]\times u \qquad \text{[for small } u\text{]} \\ &= l_x\mu(x)u. \end{aligned}$$

Therefore, for small u, the expected number of deaths in the age interval $(x, x+u)$ is approximately $l_x\mu(x)u$. Hence, $l_x\mu(x)$ can be interpreted as the rate of (expected) deaths at age x per l_0 newborns. Because of this interpretation, the plot of $l_x\mu(x)$ against x, is often called the *curve of deaths*.

Examine peculiarities of the curve of deaths based on the mortality of the ELT-12-Males (Lecture 11 hand out).

Laws of Mortality

Many attempted to describe mortality experienced by human or animal populations by a mortality law, i.e. by a mathematical expression for $\mu(x)$ (or $s(x)$). A few famous examples are:-

de Moivre's Law (1725)	$s(x) = k(\omega - x)$, $0 \le x \le \omega$
Gompertz' Law (1825)	$\mu(x) = bc^x$, $x \ge 0$
Makeham's Law (1860)	$\mu(x) = a + bc^x$, $x \ge 0$
Makeham's 2nd Law (1889)	$\mu(x) = a + hx + bc^x$, $x \ge 0$

The most famous law of mortality is that of Makeham. In this law the term a represents the mortality due to accident and the term bc^x represents the mortality due to aging. When $c = 1$ Makeham's law assumes a constant force of mortality, hence exponentially distributed X, the time-until-death for a newborn.

When a small range of ages is concerned it is often possible to find such values of a, b and c that Makeham's law fits well the observed rates of mortality. However, neither this nor any other law of mortality represent the observed mortality of human populations over a large interval of ages. And it seems unlikely that there exists a universal law expressing the mortality of human populations by the way of a simple formula.

As already mentioned, given force of mortality one can find the corresponding survival function and vice versa. For instance, for Makeham's law:

$$s(x) = e^{-\int_0^x \mu(u)\,du} = e^{-ax-b\frac{c^x-1}{\ln c}} = e^{-ax-m(c^x-1)}, \text{ where } m = \frac{b}{\ln c}.$$