Two Sorts of Interest

In the course of business and otherwise people need or like to borrow money. The reward to the money lender is called *interest*. The original loan is called *capital* or *principal*.

Interest is expressed as a rate in two senses:- per unit capital and per unit time.

5% p.a. interest means £5 per £100 of the principal per year.

The interest rate on a transaction is affected by

- the market rate for similar loans;
- the risk involved in the use to which the borrower puts the money (cf. mortgage loan rates and unsecured personal loan rates);
- the anticipated rate of appreciation or depreciation in the value of the currency in which the transaction is carried out (e.g., in times of high inflation the interest is higher).

There are two sorts of interest:- simple and compound.

Take the simplest case: savings accounts. Suppose you deposit £100 in a savings account which pays 5% p.a. At the end of the year, you can close your account and withdraw £105, £100 return of the original deposit + £5 interest.

Simple Interest just adds £5 each year to your original deposit of £100. The general formula for the accumulated amount is

$$C(1+ni) \tag{1}$$

where C = principal invested;
i = rate of simple interest;
n = units of time. Usually, but not always, years.

Expression (1) applies for all non-negative values of n. The normal commercial practice in relation to fractional periods of a year is to pay interest on a pro rata basis. For an account of duration of less than one year it is usual to allow for the actual number of *days* the account is held.

The financial press quote different rates per annum for different values of n up to 6 months or a year.

Example. (Overnight money, $n = \frac{1}{365}$)

Overnight money pays one day's interest according to the formula $C(1+\frac{i}{365})$, where i is the called nominal rate per annum convertible daily

What happens when n is greater than 1 year? Suppose a savings account offers 5% p.a. simple interest. You open account A and deposit £100. In two years' time your money grows to

$$£100(1+2\times 0.05) = £110.$$

Can you do better?

Yes, you may close account A after one year, at which time you will withdraw £105 [see (1)]. Then place this sum on deposit in a new account, say B. When you close account B after one further year, the sum withdrawn is £105(1+1× 0.05)=£110.25. Notice that

$$105(1+1\times 0.05) = [100(1+1\times 0.05)](1+2\times 0.05) = 100(1+1\times 0.05)^2.$$

The difference between accounts A and B is that, effectively, account B pays interest on the interest already earned.

When interest is paid on the previous interest accrued but not withdrawn, this is called *Compound Interest*.

For n an integer, the obvious revision of (1) for compound interest is

$$C(1+i)^n. \tag{2}$$

Indeed, if A_n is the accumulation after n years then $A_n = A_{n-1}(1+i) = A_{n-2}(1+i)^2 = \ldots = A_0(1+i)^n$, where A_0 is the capital invested initially, i.e. C.

From now on consider compound interest only.

Examples of different kinds of transactions:

- payment of a series of premiums throughout a given time period in return for a lump sum at the end of the period;
- mortgage loans, i.e. loans which are made for the specific purpose of house purchase (The property to be purchased usually acts as security for the loan);
- fixed interest securities, such as regular income payments throughout a given time period and an additional lump sum at the end of the period in return for a one-off payment.

Compound Interest can assist in comparing the merits of transactions.

Example.

You have £10,000 to invest now and are being offered £22,500 after ten years as the return from the investment. The market rate is 10% p.a. (compound interest). Ignoring complications such as the effect of taxation, the reliability of the company offering the contract, etc., do you accept the investment? The answer is 'no' because $10000(1+0.1)^{10} = 25937 > 22500$.

SUMMARY.	
Accumulation of C units of money, simple interest:	$C(1+ni)$
Accumulation of C units of money, compound interest:	$C(1+i)^n$