Actuarial Mathematics Sample Test Solutions

1. If the nominal rate of interest per annum is 6% when interest is compounded monthly, find the annual equivalent rate of interest.

   $i^{(12)} = 0.06$ and hence $i = (1+i^{(12)}/12)^{12} - 1 = (1.005)^{12} - 1 = 0.0616778$. So AER is 6.16778% .

   How much interest should be paid in arrears for the use of £1,000 over a one month period?

   $£1000 i^{(12)}/12 = £1000 \times 0.005 = £5$

   Find the present value of £5000 which is to be received in 5 years time.

   $£5000 V^5 = £5000 \frac{1}{(1+i^{(12)}/12)^{60}}$ Using either method gives $£3,706.86$

2. £1,000 is invested for 3 years. For the first 6 months the interest rate is 0.5% per month. For the remaining period the APR is 5%. How much interest will have accrued by the end of the 3 year period.

   (Find accumulated amount then take off the principal)

   $£1000(1.005)^6(1.05)^{2.5} - £1000 = £1,164.04 - £1000 = £164.04$

3. John Brown takes out a loan for $£100,000$ to purchase a property. The loan is to be repaid over a 25 year period by making equal annual payments in arrears. Find the annual payment if the APR is 10%.

   Annual payment $£P$ such that $100,000 = P a_{\overline{25|}}$. Hence

   $$P = \frac{100000(1-V)}{V(1-V^{25})} = \frac{10000}{\left(1-\left(\frac{10}{11}\right)^{25}\right)} = 11,016.8072$$

   Hence the annual payment is $£11,016.81$

   He sells his property and repays the outstanding amount of the loan at the time that the 10^{th} annual payment is due (but has not been paid). How much does he need to pay at that time?

   Either: use (present value of loan minus repayments made) times $(1+i)^{10}$ to give value in 10 years of outstanding amount, i.e.

   $$£\left(100,000 - P a_{\overline{9|}}\right)(1+i)^{10}$$

   Or: use value at time 10 of future payments, i.e. $£P\ddot{a}_{\overline{16|}}$

   Both methods give $£94,811.51$

4. The survival function $S(x) = e^{-x/100}$ for $x > 0$. Find the instantaneous death rate $\mu(x)$.

   $$\mu(x) = -S'(x)/S(x) = \frac{(1/100)e^{-x/100}}{e^{-x/100}} = 1/100.$$

   If $K(x)$ is the curtate further lifetime of a life aged x, find $P(K(x) = k)$ and give the range.

   $$P(K(x)=k) = \frac{S(x+k) - S(x+k+1)}{S(x)} = \frac{e^{-(x+k)/100} - e^{-(x+k+1)/100}}{e^{-x/100}} = e^{-k/100}(1-e^{-1/100})$$

The range is all non-negative integers k.

Find ${}_{30}q_{20}$.

$$1-\frac{S(50)}{S(20)}=1-\frac{e^{-50/100}}{e^{-20/100}}=1-e^{-0.3}=0.2592$$

5 Use the life table ELT12 to obtain the following results:

(a) Calculate the probability that a newborn survives to age 21.

$${}_{21}p_0=\frac{l_{21}}{l_0}=\frac{96178}{100,000}=0.96178$$

(b) Calculate the expected number surviving to age 30 out of 1000 men aged 20.

Number surviving is binomial parameters 1000 and ${}_{10}p_{20}$ Hence mean number is

$$1000{}_{10}p_{20}=1000\times\frac{l_{30}}{l_{20}}=1000\frac{95265}{96293}=989.3242$$

(c) Let x and n be integers and let $0<t<1$. Use linear interpolation of the survival function between integer ages to show that ${}_{n|t}q_x=t\times{}_{n|1}q_x$.

$${}_{n|t}q_x=\frac{l_{x+n}-l_{x+n+t}}{l_x}=\frac{l_{x+n}-((1-t)l_{x+n}+tl_{x+n+1})}{l_x}=t\times\frac{l_{x+n}-l_{x+n+1}}{l_x}=t\times{}_{n|1}q_x$$

Find the probability that a life who has just reached 60 survives to his 65^{th} birthday but dies within the next 6 months.

$${}_{5|0.5}q_{60}=0.5\times{}_{5|1}q_{60}=0.5\times\frac{l_{65}-l_{66}}{l_{60}}=0.5\times\frac{68490-65991}{78924}=0.01583$$