Queen Mary
University of London

MAS224, Actuarial Mathematics

Mid-term Test

3 p.m., 20 February, 2008

The duration of this test is 45 minutes. The paper has 6 questions carrying 35 marks in total. Answer all questions and write your answers in the space provided.

Do not start reading this paper until instructed by the invigilators.

Table ELT12 is provided on the last page.

Write your name and student number below.

Student Number:

Name:

Present values of annuities-certain:

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{1-v} \qquad a_{\overline{n}|} = v\ddot{a}_{\overline{n}|}$$

$$\ddot{a}^{(p)}_{\overline{n}|} = \frac{1}{p}\left[\frac{1-v^n}{1-v^{\frac{1}{p}}}\right] \qquad a^{(p)}_{\overline{n}|} = v^{\frac{1}{p}}\ddot{a}^{(p)}_{\overline{n}|}.$$

1. (5 marks) If the rate of interest is 0.75% *per month* when interest is compounded monthly, find the nominal rate of interest and the annual equivalent rate (AER). Give your answers as percentages and to 2 decimal places.

How much interest should be paid in arrears for the use of £1,000 for one year? Give your answer to the nearest penny.

How much interest should be paid in advance for the use of £1,000 for one year? Give your answer to the nearest penny.

2. **(4 marks)** A loan of £10,000 is to be repaid by payments of £P per month in arrears for 10 years. Find the monthly payment if the annual interest rate is 7%. Give your answer to the nearest penny.

3. **(3 marks)** Suppose that £1,000 is to be invested annually in advance for 5 years in an account paying interest at 5% per annum. Find the total accumulated value of this investment just after the final payment has been made. Give your answer to the nearest penny.

4. **(3 marks)** Joe Bloggs has a debt of £2,000 due 2 years from now and another one of £1,000 due 3 years from now. If Joe is allowed to discharge these debts by a single payment of £P in one year's time, what should this payment be if the interest is charged at 10% per annum? Give your answer to the nearest penny.

5. (8 marks) Suppose that beetles in a particular population are certain to die before they reach age 4 and their mortality is described by the survival function $s(x) = \frac{4-x}{4}$ for $0 \leq x \leq 4$.

Find the instantaneous death rate (force of mortality) at age 2. Use this result to obtain an approximation for the expected number of beetles out of 1,000 alive at age 2 who die within one week of reaching that age (i.e. age 2). Give your answer to the nearest integer.

If $K(x)$ is the curtate further lifetime at age x, find the probability mass function of $K(2)$. Hence find e_2, the curtate further expectation of life at age 2, for this population of beetles.

6. (12 marks) Use the life table ELT12 to obtain the following results:

(a) Calculate to 4 decimal places the probability that a life aged 60 survives to age 80.

(b) Calculate the expected number of deaths between age 60 and 65 out of 1000 newborns. Give your answer to the nearest integer.

(c) Express ${}_tq_x$ in terms of the survival function $s(x)$.

Let x be an integer and let $0 < t < 1$. Use linear interpolation of the survival function between integer ages to show that ${}_tq_x = t \times q_x$.

Find the probability that a life who has just reached his 65^{th} birthday dies within the next 9 months. Give your answer to 4 significant figures.

END