MAS224, Actuarial Mathematics: Interpolation on life table functions

Linear interpolation: within each year of age $[x, x+1]$, $x = 0, 1, 2, \ldots$, the values of $s(y)$ and l_y, $x \le y \le x+1$, are interpolated from the values of these functions at exact ages x and $x+1$ as follows:

$$\begin{aligned} s(x+t) &\approx (1-t)s(x) + ts(x+1), \qquad (0 \le t \le 1) \\ l_{x+t} &\approx (1-t)l_x + tl_{x+1}. \end{aligned}$$

Linear interpolation is consistent with the assumption of a uniform distribution of deaths within each year of age.

Estimating the force of mortality at age x:

Force of mortality cannot be directly observed. Its value at exact age x has to be calculated from the values of other life-table functions.

Linear interpolation is not suitable for this purpose as it gives two different values of $\mu(x)$, one when it is used over the interval $[x-1, x]$ and the other when it is used over $[x, x+1]$. Because of this, other approximations are used. Four most common approximations are:

(1) Based on $p_x = e^{-\int_0^1 \mu(x+t)dt}$.

$$\int_0^1 \mu(x+t)dt = -\ln p_x \qquad \therefore \quad \mu(x+\frac{1}{2}) \approx -\ln p_x$$

(2) Based on $p_{x-1}p_x = {}_2p_{x-1} = e^{-\int_{x-1}^{x+1} \mu(u)du} = e^{-\int_{-1}^{1} \mu(x+t)dt}$.

$$-\ln(p_{x-1}p_x) = \int_{-1}^{1} \mu(x+t)dt \approx 2\mu(x) \qquad \therefore \quad \mu(x) \approx -\frac{1}{2}\Big(\ln p_x + \ln p_{x-1}\Big)$$

(3) Based on the assumption that l_y is a quadratic polynomial in y in the interval $[x-1, x+1]$.

$$\mu(x) \approx \frac{l_{x-1} - l_{x+1}}{l_x}.$$

(4) Based on the assumption that l_y is a quartic polynomial in y in the interval $[x-2, x+2]$.

$$\mu(x) \approx \frac{8(l_{x-1} - l_{x+1}) - (l_{x-2} - l_{x+2})}{12 l_x}.$$

Worked Example: Using approximation (4) and the values of l_x for ages 38,39,40,41,42 from the English Life Table No. 12 – Males, estimate $\mu(40)$

$$\begin{aligned} \mu(x) &\approx \frac{8(l_{x-1} - l_{x+1}) - (l_{x-2} - l_{x+2})}{12 l_x} \\ &= \frac{8(93991 - 93570) - (94176 - 93328)}{12 \times 93790} = 0.00224. \end{aligned}$$

The obtained value coincides with the value of μ_{40} in the table.