Q1 (a) $\quad 1+i = \left(1+\frac{i^{(p)}}{p}\right)^p$

$i^{(12)} = 0.08$, hence $i = \left(1+\frac{0.08}{12}\right)^{12} - 1 = 0.0830$ (to 4 d.p.)

Effective annual rate 8.30%

(i) $i^{(12)} = 0.08$, hence the monthly rate is $\frac{0.08}{12}$.

$1000 \times \frac{0.08}{12} = 6.67$ (to 2 d.p.)

Have to pay £6.67 in arrears for use of £1000 over 1 month

(ii) Effective annual rate is 8.30%, hence should pay £83 for use of £1000 over one year.

(b) $(1+i)^{10}\, 1200\, a^{(12)}_{\overline{10|}}$ at $i=0.1$

$\cancel{(1.1)^{10}\, 1200\, \frac{v^{\frac{1}{12}}}{12(}}$ $\qquad a^{(12)}_{\overline{10|}} = \frac{v^{\frac{1}{12}}(1-v^{10})}{12(1-v^{1/12})}$

$$1200(1.1)^{10}\, \frac{\left(\frac{1}{1.1}\right)^{\frac{1}{12}}\left(1-\left(\frac{1}{1.1}\right)^{10}\right)}{12\left(1-\left(\frac{1}{1.1}\right)^{1/12}\right)} = 19986.37 \text{ (to 2 d.p.)}$$

will accumulate to £19,986.37

(c) $\quad d = \frac{i}{1+i}$

i is the interest to be paid in arrears for use of 1 unit of money over one year. d is the interest to be paid in advance. So d must be the present value of i due in one year, i.e. $d = iv$. This is precisely the relation $d = i/(1+i)$.

If $i = 0.12$ then $d = \frac{0.12}{1.12} = 0.10714$ (5 d.p.), hence £107.14 should be paid in advance for use of £1000 over one year

The interest to be paid in arrears for use of £1000 over three years is £1000 $((1+i)^3-1)$. The equivalent payment in advance should be of £1000 $\frac{(1+i)^3-1}{(1+i)^3}$.

$$1000\,\frac{(1.12)^3-1}{(1.12)^3} = 1000\left(1-\left(\frac{1}{1.12}\right)^3\right) = 288.22 \text{ (to 2d.p.)}$$

£288.22 to be paid in advance.

(d) Annual payment £P.

Equation of value $\quad P a_{\overline{10}|} = 10000$ at 10%

$$P = \frac{10000}{a_{\overline{10}|}} = \frac{10000}{v\,\frac{1-v^{10}}{1-v}} = \frac{10000\left(1-\frac{1}{1.1}\right)}{\frac{1}{1.1}\left(1-\left(\frac{1}{1.1}\right)^{10}\right)}$$

$P = 1627.45$ (to 2d.p.)

John Brown is paying £1,627.45 annually.

Schedule of payments:

	Payment (£)	Interest paid (£)	Capital paid (£)	Amount Outst (£)
0				10,000
1	1627.45	1000	627.45	9,372.55
2	1627.45	937.255	690.195	8,682.355
3	1627.45	868.2355	759.2145	7,923.1405

The amount outstanding is £7,923.14 (to 2d.p.)

Continue the schedule of payments but now at 8% p.a. I carry out calculations to 2 d.p.

	Payment (£)	Interest paid (£)	Capital paid (£)	Amount Outst (£)
3	1627.45			7923.14
4	1627.45	633.85	993.60	6929.54
5	1627.45	554.36	1073.09	5856.45
6	1627.45	468.52	1158.93	4697.52
7	1627.45	375.80	1251.65	3445.87
8	1627.45	275.67	1351.78	2094.09
9	1627.45	167.53	1459.92	634.17
10	634.17	50.73		

It will take 7 years, with the final payment being of £634.17 + £50.73 = £684.90 (2d.p.)

Q2 (a)

$$ {}_n p_x = \frac{s(x+n)}{s(x)}, \quad {}_n q_x = \frac{s(x)-s(x+n)}{s(x)}, $$

$$ {}_{n|m} q_x = \frac{s(x+n)-s(x+n+m)}{s(x)}, $$

$$ l_x = l_0\, s(x), \quad {}_n d_x = l_x - l_{x+n} $$

(b) This prob. is $\cancel{{}_{34}q_{21}}$ ${}_{44}q_{21}$

$$ \cancel{{}_{34}p_{21} = \frac{l_{65}}{l_{21}} = \frac{68490}{96178} = 0.71} \qquad \cancel{{}_{34}q_{21} = \frac{l_{21}-l_{65}}{l_{21}}} $$

$$ {}_{44}q_{21} = 1 - \frac{l_{65}}{l_{21}} = 1 - \frac{68490}{96178} = \underline{0.2879} \text{ (to 4d.p.)} $$

(c) This prob is ${}_{44|1}q_{21}$

$$ {}_{44|1}q_{21} = \frac{l_{65}-l_{66}}{l_{21}} = \frac{68490-65991}{96178} = \underline{0.0260} \text{ (to 4d.p.)} $$

(d) This number is $1000\ {}_{45}p_{20}$

$$1000\,{}_{45}p_{20} = 1000\,\frac{\ell_{65}}{\ell_{20}} = 1000\,\frac{68490}{96293} = \underline{\underline{711.2667}} \text{ (to 4d.p.)}$$

(e) linear interpolation on $S(x+t)$:

$$S(x+t) \approx (1-t)S(x) + tS(x+1), \quad \text{integer } x,\ 0<t<1$$

$$\begin{aligned}
{}_t d_x = \ell_x - \ell_{x+t} &= \ell_0 S(x) - \ell S(x+t) \\
&\approx \ell_0 (S(x) - (1-t)S(x) - tS(x+1)) \\
&= \ell_0\, t(S(x) - S(x+1)) \\
&= t(\ell_0 S(x) - \ell_0 S(x+1)) = t d_x
\end{aligned}$$

This number is ${}_{\frac{1}{2}}d_{65}$ (as $100000 = \ell_0$)

$${}_{\frac{1}{2}}d_{65} \approx \left(\tfrac{1}{2}\right) d_{65} = 2499/2 = \underline{1249.5}$$

(f) $K(x)$ is the number of complete further years to be lived by a life age x.

$$P(K(x)=k) = P(k \le T(x) < k+1) = {}_{k|1}q_x = \frac{\ell_{x+k} - \ell_{x+k+1}}{\ell_x}$$

$$= \frac{\ell_0 S(x+k) - \ell_0 S(x+k+1)}{\ell_0 S(x)} = \frac{S(x+k) - S(x+k+1)}{S(x)}$$

$$e_x = E(K(x)) = \sum_{k=0}^{\infty} k\, P(K(x)=k)$$

$$= \sum_{k=0}^{\infty} k\, \frac{S(x+k) - S(x+k+1)}{S(x)} = \frac{1}{S(x)} \sum_{k=0}^{\infty} k\, S(x+k) -$$

$$\frac{1}{S(x)} \sum_{k=0}^{\infty} k\, S(x+k+1) = \frac{1}{S(x)} \sum_{k=0}^{\infty} k\, S(x+k) -$$

$$\frac{1}{S(x)} \sum_{k'=1}^{\infty} (k'-1)\, S(x+k') = \frac{1}{S(x)} \sum_{k=1}^{\infty} k\, S(x+k) -$$

$$\frac{1}{S(x)} \sum_{k=1}^{\infty} k\, S(x+k) + \frac{1}{S(x)} \sum_{k=1}^{\infty} S(x+k) = \frac{1}{\ell_x} \sum_{k=1}^{\infty} \ell_{x+k}$$

(g) $p_{[50]} = \frac{1}{2} \times (p_{50}) = \frac{1}{2} \times 0.99272 = 0.49636$

$p_{[50]+1} = \frac{3}{4} \times (p_{51}) = \frac{3}{4} \times 0.99177 = 0.7438275$

Select period is 2 yrs, therefore

$$p_{[50]+1} = \frac{l_{[50]+2}}{l_{[50]+1}} = \frac{l_{52}}{l_{[50]+1}}$$

$$p_{[50]} = \frac{l_{[50]+1}}{l_{[50]}}$$

$$\therefore \quad l_{[50]+1} = \frac{l_{52}}{p_{[50]+1}} = \frac{88693}{0.7438275} = \underline{\underline{119,238.6676}}$$

$$l_{[50]} = \frac{l_{[50]+1}}{p_{[50]}} = \frac{119238.6676}{0.49636} = \underline{\underline{240,226.1817}}$$

(i) $\mathring{e}_{[50]}$ wanted

$$\mathring{e}_{[50]} \approx e_{[50]} + \frac{1}{2}$$

$$e_{[50]} = \frac{1}{l_{[50]}} \left(l_{[50]+1} + l_{[50]+2} + l_{[50]+3} + \dots \right)$$

$$= \frac{1}{l_{[50]}} \left(l_{[50]+1} + l_{52} + l_{53} + \dots \right)$$

$$= \frac{l_{[50]+1}}{l_{[50]}} + \frac{1}{l_{[50]}} \frac{l_{51}}{l_{51}} \left(l_{52} + l_{53} + \dots \right)$$

$$= p_{[50]} + \frac{l_{51}}{l_{[50]}} \times e_{51} = p_{[50]} + \frac{l_{51}}{l_{[50]}} \left(\mathring{e}_{51} - \frac{1}{2} \right)$$

$$\therefore \quad \mathring{e}_{[50]} \approx p_{[50]} + \frac{l_{51}}{l_{[50]}} \left(\mathring{e}_{51} - \frac{1}{2} \right) + \frac{1}{2}$$

$$= 0.49636 + \frac{89429}{240226.1817} (21.84 - 0.5) + 0.5$$

$$= \underline{\underline{8.44}} \text{ (to 2 d.p.)}$$

(ii) $\mathring{e}_{[50]+1}$ wanted

$$\mathring{e}_{[50]+1} \approx \cancel{e_{[50]}}\ e_{[50]+1} + \frac{1}{2}$$

$$e_{[50]+1} = \frac{1}{l_{[50]+1}} \left(l_{[50]+2} + l_{[50]+3} + \dots \right)$$

$$= \frac{1}{l_{[50]+1}} \left(l_{52} + l_{53} + \dots \right)$$

$$= \frac{l_{51}}{l_{[50]+1}} \frac{1}{l_{51}} \left(l_{52} + l_{53} + \dots \right) = \frac{l_{51}}{l_{[50]+1}} e_{51}$$

$$= \frac{l_{51}}{l_{[50]+1}} \left(\mathring{e}_{51} - \frac{1}{2} \right)$$

$$\therefore \quad \mathring{e}_{[50]+1} \approx \frac{l_{51}}{l_{[50]+1}} \left(\mathring{e}_{51} - \frac{1}{2} \right) + \frac{1}{2}$$

$$= \frac{89429}{119238.6676} \left(21.84 - 0.5 \right) + 0.5$$

$$= 16.50 \text{ (to 2d.p.)}$$

Q3 (a) The cost is £100,000 $A_{[40]:\overline{20|}}$

By conversion relationship $A_{[40]:\overline{20|}} = 1 - d\,\ddot{a}_{[40]:\overline{20|}}$ and, also, $\ddot{a}_{[40]:\overline{20|}} = \dfrac{N_{[40]} - N_{[40]+20}}{D_{[40]}} = \dfrac{N_{[40]} - N_{60}}{D_{[40]}}$

$$\therefore\ 100{,}000\left(1 - d\,\frac{N_{[40]} - N_{60}}{D_{[40]}}\right) =$$

$$100000\left(1 - \frac{4}{104}\,\frac{131995.19 - 35841.261}{6981.5977}\right) = 47028.92$$

(to 2 d.p.)

The cost is £47,028.92

(b) £P - annual premium.

Equation of value: cost $= P\,\ddot{a}_{[40]:\overline{20|}}$

$$\therefore\ P = \frac{47{,}028.92}{\ddot{a}_{[40]:\overline{20|}}} = \frac{47028.92}{N_{[40]} - N_{60}} \times D_{[40]}$$

$$= \frac{47028.92 \times 6981.5977}{131995.19 - 35841.261} = 3414.70 \text{ (to 2 d.p.)}$$

Annual premium of £3,414.70

(c) Surrender value = policy value (ignoring expenses)

Policy value = E(value of benefit payments at age 50 − value of future ~~pay~~ premiums at age 50)

$$= \left(A_{50:\overline{10|}} - P_{[40]}\,\ddot{a}_{50:\overline{10|}}\right) \times 100000$$

$$= \left(\left(1 - d\,\ddot{a}_{50:\overline{10|}}\right) - \left(\frac{1}{\ddot{a}_{[40]:\overline{20|}}} - d\right)\ddot{a}_{50:\overline{10|}}\right) \times 100000$$

$$= \left(1 - \frac{\ddot{a}_{50:\overline{10|}}}{\ddot{a}_{[40]:\overline{20|}}}\right) \times 100000$$

~~Note~~ with

$$100{,}000\left(1-\frac{\ddot{a}_{50:\overline{10|}}}{\ddot{a}_{[40]:\overline{20|}}}\right)=100{,}000\left(1-\frac{\frac{N_{50}-N_{60}}{D_{50}}}{\frac{N_{[40]}-N_{60}}{D_{[40]}}}\right)$$

$$=100{,}000\left(1-\frac{(N_{50}-N_{60})\,D_{[40]}}{(N_{[40]}-N_{60})\,D_{50}}\right)$$

$$=100000\left(1-\frac{(73567.136-35841.261)\times 6981.5977}{(131995.19-35841.261)\times 4597.0607}\right)$$

$= 40413.64$ (to 2 d.p.)

Surrender value £40,413.64

If this is used to buy fully paid up whole life assurance then can find the ~~[illegible]~~ sum assured C from the equation of value

$$40413.64 = C A_{50} = C(1-d\,\ddot{a}_{50})$$

$$\therefore\ C=\frac{40413.64}{1-d\,\frac{N_{50}}{D_{50}}}=\frac{40413.64}{1-\frac{4}{104}\times\frac{73567.136}{4597.0607}}$$

$= 105107.85$ (to 2 d.p.)

Surrender value will buy £105107.85 of ~~wholly~~ fully paid up whole life assurance.

(d)

$$Y=\begin{cases}\ddot{a}_{\overline{K(x)+1|}} & \text{if } 0\le K(x)\le n-1\\ \ddot{a}_{\overline{n|}} & \text{if } K(x)\ge n\end{cases}$$

$$E(Y)=\sum_k y_k P(Y=y_k)=\sum_{k=0}^{n-1}\ddot{a}_{\overline{k+1|}}\,P(K(x)=k)+\ddot{a}_{\overline{n|}}\,P(K(x)\ge n)$$

Using $P(K(x)=k) = {}_k p_x - {}_{k+1}p_x$, $P(K(x)\ge n) = P(T(x)\ge n) = {}_n p_x$

and $\ddot{a}_{\overline{k+1|}} = \ddot{a}_{\overline{k|}} + v^k$

$$E(Y) = \sum_{k=0}^{n-1} \ddot{a}_{\overline{k+1}|} ({}_kp_x - {}_{k+1}p_x) + \ddot{a}_{\overline{n}|}\, {}_np_x$$

$$= \sum_{k=0}^{n-1} \ddot{a}_{\overline{k+1}|}\, {}_kp_x - \sum_{k=0}^{n-1} \ddot{a}_{\overline{k+1}|}\, {}_{k+1}p_x + \ddot{a}_{\overline{n}|}\, {}_np_x$$

$$= \sum_{k=0}^{n-1} \ddot{a}_{\overline{k+1}|}\, {}_kp_x - \sum_{k'=1}^{n} \ddot{a}_{\overline{k'}|}\, {}_{k'}p_x + \ddot{a}_{\overline{n}|}\, {}_np_x \quad ; \quad k' = k+1$$

$$= \sum_{k=0}^{n-1} (v^k + \ddot{a}_{\overline{k}|})\, {}_kp_x - \sum_{k'=1}^{n-1} \ddot{a}_{\overline{k'}|}\, {}_{k'}p_x$$

$$= \sum_{k=0}^{n-1} v^k\, {}_kp_x + \sum_{k=0}^{n-1} \ddot{a}_{\overline{k}|}\, {}_kp_x - \sum_{k=1}^{n-1} \ddot{a}_{\overline{k}|}\, {}_kp_x$$

the last two sums above cancel each other because $\ddot{a}_{\overline{0}|} = 0$ and the zero-term $(k=0)$ is irrelevant.

$$\therefore\ E(Y) = \sum_{k=0}^{n-1} v^k\, {}_kp_x = \sum_{k=0}^{n-1} v^k \frac{l_{x+k}}{l_x} = \sum_{k=0}^{n-1} \frac{v^{x+k} l_{x+k}}{v^x l_x}$$

$$E(Y) = \sum_{k=0}^{n-1} \frac{D_{x+k}}{D_x} = \frac{1}{D_x}\left(\sum_{k=0}^{\infty} D_{x+k} - \sum_{k=0}^{\infty} D_{x+n+k} \right)$$

$$= \frac{N_x - N_{x+n}}{D_x}$$

Q4. (a) $P(T(x) > t) = {}_tp_x = \dfrac{s(x+t)}{s(x)}$; $\mu(x) = -\dfrac{1}{s(x)} \dfrac{d}{dx} s(x)$

p.d.f. of $T(x)$: $f_{T(x)}(t) = \dfrac{d}{dt} F_X(t) = \dfrac{d}{dt}\left(1 - P(T(x) > t)\right)$

$$= -\frac{s'(x+t)}{s(x)}$$

$$\therefore\ f_{T(x)}(t) = -\frac{s'(x+t)}{s(x)} \times \frac{s(x+t)}{s(x+t)} = -\frac{s'(x+t)}{s(x+t)} \frac{s(x+t)}{s(x)}$$

$$= \mu(x+t) \frac{s(x+t)}{s(x)}$$

If $s(x) = e^{-x/50}$, $x \geq 0$, then $s'(x) = -\frac{1}{50} e^{-x/50}$

and $\mu(x) = -\dfrac{s'(x)}{s(x)} = \dfrac{1}{50} \dfrac{e^{-x/50}}{e^{-x/50}} = \dfrac{1}{50}$

instantaneous death rate (a.k.a. force of mortality) is $\frac{1}{50}$ (per unit time, per ~~each~~ individual age x)

$$\mu(x+t)=\frac{1}{50}\quad,\quad \frac{S(x+t)}{S(x)}=\frac{e^{-(x+t)/50}}{e^{-x/50}}=e^{-t/50}$$

$$\therefore\quad f_{T(x)}(t)=\mu(x+t)\frac{S(x+t)}{S(x)}=\frac{1}{50}e^{-t/50},\ t\geq 0.$$

$$\mathring{e}_x=E(T(x))=\int_0^\infty t\,f_{T(x)}(t)\,dt$$

$$=\frac{1}{S(x)}\int_0^\infty S(x+t)\,dt\quad\text{provided } S(x+t)\to 0 \text{ as } t\to\infty$$

$$\mathring{e}_x=\frac{1}{\cancel{\frac{1}{50}}e^{-x/50}}\int_0^\infty \cancel{\frac{1}{50}}e^{-(x+t)/50}\,dt=\int_0^\infty e^{-t/50}\,dt$$

$$=50\int_0^\infty e^{-t/50}\,d(t/50)=-50\left[e^{-t/50}\right]_{t=0}^{t=\infty}=50$$

$\underline{\underline{\mathring{e}_x=50 \text{ all } x}}$ (Will accept $\mathring{e}_x=E(T(x))$ & $=E(\mathrm{Exp}(\frac{1}{50}))=50$.)

(b) ~~$Y=\{v^{T(x)}$~~

$$Y=\begin{cases} Cv^n, & \text{if } T(x)\geq n\\ 0 & \text{otherwise}\end{cases}$$

$$E(Y)=Cv^n P(T(x)\geq n)+0\times P(T(x)<n)$$

$$=Cv^n\,{}_np_x=Cv^n\frac{S(x+n)}{S(x)}$$

$$\mathrm{var}(Y)=E(Y^2)-E(Y)^2=C^2v^{2n}P(T(x)\geq n)-C^2v^{2n}\left(\frac{S(x+n)}{S(x)}\right)^2$$

$$=C^2v^{2n}\frac{S(x+n)}{S(x)}-\left(\frac{S(x+n)}{S(x)}\right)^2$$

$$=C^2v^{2n}\frac{S(x+n)}{S(x)}\left(1-\frac{S(x+n)}{S(x)}\right)$$

20 year endowment policy to a newborn with benefit £ 10,000 at 8% p.a.

$$E(Y) = \frac{10000\, v^{20}\, l_{20}}{l_0} = \frac{10{,}000 \left(\frac{1}{1.08}\right)^{20} 96293}{100{,}000} = \underline{\underline{2065.95}} \text{ (2d.p.)}$$

$$\operatorname{var}(Y) = \frac{(10{,}000)^2 v^{40} l_{20}}{l_0}\left(1 - \frac{l_{20}}{l_0}\right) =$$

$$= \frac{(10{,}000)^2 \left(\frac{1}{1.08}\right)^{40} 96293}{100000}\left(1 - \frac{96293}{100000}\right)$$

$$= \underline{\underline{164311.17}} \text{ (to 2d.p.)}$$

If Z is the total expected present value of benefit payments to 100 newborns, i.e. $Z = Y_1 + Y_2 + \dots + Y_{100}$, then

$E(Z) = 100\, E(Y) = 206595$ and

$\operatorname{Var}(Z) = 100 \operatorname{Var}(Y) = 16431117$

The minimum amount h (in pounds) to be invested is determined by the equation $P(Z \le h) = 0.95$

$$\frac{Z - E(Z)}{\sqrt{\operatorname{Var}(Z)}} \sim N(0,1)$$

$\therefore$ have the equation $P\left(\frac{Z - E(Z)}{\sqrt{\operatorname{Var}(Z)}} \le \frac{h - E(Z)}{\sqrt{\operatorname{Var}(Z)}}\right) = 0.95$

i.e. $$\frac{h - E(Z)}{\sqrt{\operatorname{Var}(Z)}} = 1.6449$$

$$h = E(Z) + \sqrt{\operatorname{Var}(Z)} \times 1.6449$$

$$= 206595 + \sqrt{16431117} \times 1.6449 = 213262.65 \text{ (2d.p.)}$$

$\therefore$ the minimum amount to be invested per policy is

£ 2,132.63 (to 2 d.p.)

Q5. Let $Y_x(t)$ be the number of females age x at time t, so that $n_x(t) = E(Y_x(t))$

Then $Y_{x+1}(t+1) \mid Y_x(t) \sim Bin(Y_x(t), p_x)$ and, therefore,

$$n_{x+1}(t+1) = E(Y_{x+1}(t+1)) = E(E(Y_{x+1}(t+1) \mid Y_x(t)))$$

$$= E(E(Bin(Y_x(t), p_x))) = E(\cancel{Bin}\, Y_x(t) p_x)$$

$$= n_x(t) p_x.$$

for $x = 0, 1, 2, \ldots$

Thus $\quad n_{x+1}(t+1) = n_x(t) p_x \quad x = 0, 1, 2, \ldots$

$Y_0(t+1)$ is the number of all births in the year from t to $t+1$.

$$Y_0(t+1) = \sum_x \left(\sum_{k=1}^{Y_x(t)} (\text{number of births to the } k^{th} \text{ female age } x \text{ at time } t) \right)$$

$$\therefore \quad n_0(t+1) = E(Y_0(t+1))$$

$$= \sum_x E\left(E\left(\sum_{k=1}^{Y_x(t)} (\text{no. of births to the } k^{th} \text{ female}) \mid Y_x(t)\right)\right)$$

$$= \sum_x E(Y_x(t) b_x) = \sum_x b_x n_x(t)$$

$$\therefore \quad n_0(t+1) = \sum_x b_x n_x(t).$$

(a) $N = 5000$

$$N = N(0) = A l_0 \left(\tfrac{1}{2} + \mathring{e}_0\right)$$

$$\therefore \quad A = \frac{N}{l_0\left(\frac{1}{2} + \mathring{e}_0\right)} = \frac{\cancel{5000}}{100\cancel{000}(0.5 + 68.09)}$$

$$= 0.000728969$$

Expected number of births is $n(0)$

$$n(0) = A l_0 = 72.90 \quad (2\text{ d.p.})$$

On average 72.90 births expected each year.

Expected number of aged under 21 is $N - N(21)$

$N(21) = A l_{21}\left(\frac{1}{2} + \mathring{e}_{21}\right) = (0.000728969) \times (96178) \times (0.5 + 49.63)$

$= 3514.65$

$5000 - 3514.65 = 1483.35$

Expected number aged ~~&~~ under 21 is 1483.35

(b) $b_0 = b_1 = 0$, $b_2 = 2$

$p_0 = \frac{3}{4}$ $p_1 = \frac{2}{3}$

Leslie matrix $M = \begin{pmatrix} 0 & 0 & 2 \\ \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \end{pmatrix}$

$s(0) = 1$

$s(1) = p_0 = \frac{3}{4}$ $s(2) = p_0 p_1 = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ $s(3) = 0$.

$$\sum_{x=0}^{2} b_x s(x) = b_0 s(0) + b_1 s(1) + b_2 s(2) = 0 + 0 + 2 \times \frac{1}{2} = 1.$$

Stable solution $n_x = s(x) n_0$

$\therefore$ $n_1 = \frac{3}{4} n_0$ and $n_2 = \frac{2}{3} n_0$

To find n_0, use the condition $n_0 + n_1 + n_2 = N$

$\therefore$ $n_0 + \frac{3}{4} n_0 + \frac{2}{3} n_0 = N$

$n_0 \left(1 + \frac{3}{4} + \frac{2}{3}\right) = N$ $n_0 = \frac{N}{29/12} = \frac{12N}{29}$

If the popn size is N then the stable solution is

$n_0 = \frac{12N}{29}$, $n_1 = \frac{36N}{116} = \frac{18}{58}N = \frac{9}{29}N$

$n_2 = \frac{24}{87}N$

turn over →

$$\underline{n}(0) = \begin{pmatrix} 0 \\ 1000 \\ 0 \end{pmatrix}$$

$$\underline{n}(1) = M\,\underline{n}(0) = \begin{pmatrix} 0 & 0 & 2 \\ \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1000 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{2000}{3} \end{pmatrix}$$

$$\underline{n}(2) = M\,\underline{n}(1) = \begin{pmatrix} 0 & 0 & 2 \\ \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \frac{2000}{3} \end{pmatrix} = \begin{pmatrix} \frac{4000}{3} \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{n}(3) = M\,\underline{n}(2) = \begin{pmatrix} 0 & 0 & 2 \\ \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \end{pmatrix} \begin{pmatrix} \frac{4000}{3} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1000 \\ 0 \end{pmatrix}$$

Age groups will not stalize because of oscillations

$$\underline{n}(1) = \underline{n}(4) = \underline{n}(7) = \ldots$$

$$\underline{n}(2) = \underline{n}(5) = \underline{n}(8) = \ldots$$

$$\underline{n}(3) = \underline{n}(0) = \underline{n}(6) = \ldots$$