Queen Mary
University of London

B. Sc. Examination by course unit 2009

MTH4108 Probability I

Duration: 2 hours

Date and time: 8 May 2009, 14:30

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): J Robert Johnson

Question 1 [12]

Let A and B be events with $\mathbb{P}(A) = 5/6$, $\mathbb{P}(B) = 7/12$ and $\mathbb{P}(A \cap B) = 1/2$.

(a) Calculate the following:

(i) $\mathbb{P}(A \cup B)$,

(ii) $\mathbb{P}(A^c)$,

(iii) $\mathbb{P}(A|B)$.

(b) Write down the following events in symbols:

(i) B does not occur,

(ii) B occurs but A does not,

(iii) exactly one of A and B occurs.

Question 2 [10]

(a) What does it mean for the two events E and F to be independent?

(b) A standard 6-sided fair die is rolled. Are the events "the number showing is even" and "the number showing is a multiple of 3" independent? Justify your answer.

(c) Prove that if E and F are independent events then E^c and F are independent events.

(d) What does the result you proved in part (c) imply about the situation in part (b)?

Question 3 [10]

(a) State the theorem of total probability.

I have 3 bags each containing coloured balls. The first bag contains 2 red balls; the second bag contains 2 red balls and 2 blue balls; the third bag contains 2 red balls and 4 blue balls. I pick a bag at random with all choices being equally likely and then I pick a ball from that bag, again with all choices equally likely.

(b) What is the probability that I pick a red ball?

(c) How many red balls would have to be added to the third bag to ensure that the probability of picking a red ball using this procedure is at least $2/3$?

Question 4 [8]

Let C be a continuous random variable.

(a) Say how to find the cumulative distribution function (cdf) of C if you are given its probability density function (pdf).

Suppose that the pdf of C is

$$f_C(x) = \begin{cases} 0 & \text{if } x \leq 1 \\ 1 & \text{if } 1 < x < 2 \\ 0 & \text{if } 2 \leq x \end{cases}$$

(b) Find the cdf of C.

(c) What name is given to the distribution of C?

Question 5 [12]

Let X be a discrete random variable.

(a) Define the expectation and variance of X.

Let $a, b \in \mathbb{R}$ and define the random variable Y by $Y = aX + b$.

(b) Prove that $\mathbb{E}(Y) = a\mathbb{E}(X) + b$.

(c) State and prove an analogous result for $\text{Var}(Y)$.

(d) Show that if $\text{Var}(X) \neq 0$ then a and b may be chosen so that $\mathbb{E}(Y) = 0$ and $\text{Var}(Y) = 1$.

Question 6 [8]

Let $R \sim \text{Poisson}(\lambda)$.

(a) Find $\mathbb{P}(R = 0)$.

(b) Find $\mathbb{P}(R \leq 1)$.

(c) Find $\mathbb{P}(R = 0 | R \leq 1)$.

(d) What is the conditional probability mass function of R given that $R \leq 1$?

Question 7 [20]

(a) What is meant by a Bernoulli(p) trial?

(b) Explain how the geometric distribution arises from a sequence of Bernoulli trials.

(c) Use the description of the geometric distribution you gave in part (b) to derive the probability mass function of a Geometric(p) random variable.

For the remainder of the question let X be a random variable with $X \sim$ Geometric$(1/3)$.

(d) Calculate the probability that $X < 4$.

(e) Calculate the probability that $X > \mathbb{E}(X)$.

(f) Calculate the probability that X is odd.

(g) For which n is $\mathbb{P}(X = n)$ largest?

Question 8 [20]

(a) Explain briefly what is meant by a sample space and an event.

A number is chosen from the set $\{1,2,3,4\}$ at random; then a second number is chosen at random from the remaining numbers in the set. For both choices all possibilities are equally likely.

(b) Write down the sample space for this experiment.

(c) Give an example of an event of cardinality 3 related to this experiment. Describe your event in words and as a set.

Let A be the first number chosen, B be the second number chosen, T be the sum of the two numbers which are not chosen, and P be the product of the two numbers which are not chosen.

(d) Find the probability mass functions of A and B.

(e) Find the expectations of A and B.

(f) Express T in terms of A and B and hence find the expectation of T.

(g) Why is it not possible to use the method of part (f) to find the expectation of P?

End of Paper