MTH4107 Introduction to Probability – 2010/11

Exercise Sheet 7

These questions are on discrete random variables. You should write up your solution to the starred question, Q3,* **clearly** *and hand it in during your week 10 exercise class for feedback. Put your* **full name and student number** *on the top of your solution. It is important that you make a serious attempt to do* **all** *of questions Q1-Q3 before week 10 lectures begin. Questions AQ1-AQ3 are for additional practice. You should attempt them when you have time.*
In addition to your lecture notes material relating to these questions can be found in Devore, chapter 3 sections 3.1 and 3.2, or Ross, chapter 4 sections 4.1 and 4.2

Q1. A random variable X has range $\{-2, -1, 0, 1, 2\}$ and probability mass function:

n	-2	-1	0	1	2
$P(X = n)$	$1/10$	$2/5$	$1/4$	$1/5$	$1/20$

(a) Calculate the probability of each of the following events:

(i) $X = 2$;

(ii) $X = 3$;

(iii) $X \leq 1$;

(iv) $X < 1$;

(v) $X^2 < 1$;

(b) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

(c) Let Y be a new random variable defined by $Y = X^2 + 4$. Determine the range of Y and the probability mass function of Y.

Q2. Let Z be a random variable which takes values in the set $\{0, 1, 2, 3\}$. Suppose also that $\mathbb{P}(Z = 0) = \mathbb{P}(Z = 3)$ and $\mathbb{P}(Z = 1) = \mathbb{P}(Z = 2)$.

(a) Find $\mathrm{E}(Z)$.

(b) What are the smallest and largest values that $\mathrm{Var}(Z)$ can take for a random variable of this form? (The situation where Z takes one or more of the values 0, 1, 2, 3 with 0 probability is allowed.)

(c) Suppose $\mathrm{Var}(Z) = 1$. Determine the probability mass function of Z.

Q3*. (a) A bag contains 6 red marbles and 2 blue marbles. I choose 5 at random without replacement. Let B be the number of blue marbles in my selection and R be the number of red marbles in my selection.

(i) Find the probability mass function of B and its expectation and variance.

(ii) Find the expectation and variance of R without calculating its probability mass function.

(b) Let X be a discrete random variable on a sample space S. Suppose that there exists real numbers m and M such that $m \leq X(s) \leq M$ for all $s \in S$. Prove that $m \leq \mathrm{E}(X) \leq M$. (I am asking you to give a proof for Proposition 11.2 (b) in the notes.)

AQ1. Let X be a random variable which takes values 0, 1 and 2 only. Suppose that $\mathrm{E}(X) = 3/2$ and $\mathrm{Var}(X) = 1/2$. Determine the probability mass function of X.

AQ2. Let X is a discrete random variable with $\mathrm{E}(X) = \mu$ and $\mathrm{Var}(X) = 0$. Prove that $\mathbb{P}(X = \mu) = 1$.

AQ3. Let X be a random variable taking values in the set $\{0, 1, 2, 3, \ldots, n\}$.

(a) Show that

$$\mathrm{E}(X) = \sum_{i=1}^{n} \mathbb{P}(X \geq i).$$

(b) Deduce that if $\mathrm{E}(X) < 1$ then X takes the value 0 with positive probability.