MTH4107 Introduction to Probability – 2010/11

Exercise Sheet 6

These questions are designed to help you understand the material on conditional probability. You should write up your solution to the starred question, Q4, **clearly** *and hand it in during your week 9 exercise class for feedback. Put your* **full name and student number** *on the top of your solution. It is important that you make a serious attempt to do* **all** *of questions Q1-Q3 before week 9 lectures begin. Questions AQ1-AQ3 are for additional practice. You should attempt them when you have time.*
In addition to your lecture notes material relating to these questions can be found in Devore, Chapter 2 Section 2.4 or Ross, Chapter 3 Sections 3.2 and 3.3.

Q1. I have two coins in a bag. One is fair and the other has a head on both sides. I select a coin at random from the bag, then throw it twice (without determining which coin it is). Let F be the event that the fair coin is chosen, H_1 be the event that the first throw is a head, and H_2 be the event that the second throw is a head.

(a) Calculate $\mathbb{P}(H_1 \cap F)$, $\mathbb{P}(H_2 \cap F)$, and $\mathbb{P}(H_1 \cap H_2 \cap F)$.

(b) Deduce that $H_1 \cap F$ and $H_2 \cap F$ are not independent events.

Q2. Two important players in a football team are injured, and each has probability 1/3 of recovering before the next match. The recoveries of the two players are independent of each other. If both are able to play then the team has probability 3/4 of winning the match, if only one of them plays then the probability of winning is 1/2 and if neither play the probability of winning is 1/16. What is the probability that the team wins the match?

Q3. In the 2008 US presidential election 66% of voters aged under 30 voted for Barack Obama. Also, Obama's share of the total vote was 52.6% and 18% of all voters were aged under 30.

(a) What is the probability that a voter aged 30 or over voted for Obama?

(b) What is the probability that someone who voted for Obama was aged under 30?

Q4* (Taken from 2009 late summer exam) Ten percent of the population suffer from a certain disease. Each time a person with the disease is tested for the disease, there is a 90% chance that the test will show a positive result. Each time a person without the disease is tested, there is a 1% chance of it showing a positive result. A person is chosen at random from the population.

(a) Determine the probability that the test gives a positive result for the chosen person.

(b) Determine the probability that the chosen person has the disease given that the test was positive.

(c) Determine the probability that a second test of the same person is positive given that the first test was positive.

AQ1. When I travel into work each morning I notice whether my train is late and by how much and also whether I am able to get a seat on it. Let A be the event "the train is on time", B be the event "the train is late but by not more than 15 minutes", and C be the event "I am able to get a seat". Suppose that $\mathbb{P}(A) = 1/2$, $\mathbb{P}(B) = 1/4$, $\mathbb{P}(C) = 1/3$ and $\mathbb{P}(A \cap C) = 1/4$.

(a) Find the conditional probability that the train is more that 15 minutes late given that the train is late.

(b) Find the conditional probability that I get a seat given that the train is late.

AQ2. A student observes that out of all students graduating with first class degrees in mathematics from Queen Mary , 96% of them passed the mid-term test in Probability I. He claims that since he has just passed the test he has a very good chance of getting a first.

(a) What is the most obvious mistake with this argument? Explain your answer in terms of conditional probabilties.

(b) Assume further that 80% of students pass the test and 10% of students get a first. What is the probability that a randomly chosen student who has passed the test gets a first.

AQ3. Two treatments for a disease are tested on a group of 390 patients. Treatment A is given to 160 patients of whom 100 are men and 60 are women; 20 of these men and 40 of these women recover. Treatment B is given to 230 patients of whom 210 are men and 20 are women; 50 of these men and 15 of these women recover.

(a) For which of A and B is there a higher probability that a patient chosen randomly from among those given that treatment recovers? Express this as an inequality between two conditional probabilities.

(b) For which of A and B is there a higher probability that a man chosen randomly from among those given that treatment recovers? Express this as an inequality between two conditional probabilities.

(c) For which of A and B is there a higher probability that a woman chosen randomly from among those given that treatment recovers? Express this as an inequality between two conditional probabilities.

(d) Are you surprised by your answers to the previous parts? What is going on here?