MTH4107 Introduction to Probability – 2010/11
Exercise Sheet 4

These questions are designed to help you understand the material covered in week 4 lectures. You should write up your solution to the starred question, Q4, **clearly** *and hand it in during your week 6 exercise class for feedback. Put your* **full name and student number** *on the top of your solution. It is important that you make a serious attempt to do* **all** *of questions Q1-Q4 before week 6 lectures begin. Questions AQ1-AQ3 are for additional practice. You should attempt them when you have time.*

Most of these questions are about sampling. Make sure that you read the question carefully and decide whether the sampling is with or without replacement and whether or not order is relevant. You may assume for this sheet that when a choice is described as "at random", all possibilities are equally likely. In addition to your lecture notes material relating to these questions can be found in Devore, Chapter 2 Section 2.3 or Ross, Chapter 2 Section 2.5.

Q1. When I open a bank account I am allocated a 4 digit personal identification number (PIN) at random. Find the probability of each of the following events, giving brief explanations for your answers.

(a) Every digit of my PIN is even.

(b) My PIN has no repeated digits.

(c) My PIN is palindromic (reads the same forwards as backwards).

(d) No digit of my PIN exceeds 7.

(e) The digits in my PIN are in strictly increasing order.

Q2. Each member of a squad of 18 cricketers is either a batsmen or a bowler. The squad contains 10 batsmen and 8 bowlers. An eccentric coach chooses a team by picking a random set of 11 players from the squad.

(a) What is the probability that the team is made up of 6 batsmen and 5 bowlers?

(b) What is the probability that the team contains fewer than 3 bowlers?

Q3.

(a) Let S be the set of all functions from $\{1, 2\}$ to $\{1, 2, 3\}$. Let f be an element of S chosen at random. Find $|S|$ and determine the probability that f is an injective function.

(b) Let S be the set of all functions from $\{1, 2, 3\}$ to $\{1, 2\}$. Let g be an element of S chosen at random. Find $|S|$ and determine the probability that g is an injective function.

(c) Let S be the set of all functions from $\{1, 2, \ldots, m\}$ to $\{1, 2, \ldots, n\}$. Let h be an element of S chosen at random. Find $|S|$ and determine the probability that h is an injective function.

Q4* A bag contains two copper coins, three silver coins and four gold coins.

(a) I randomly select five coins from the bag one after the other replacing each coin in the bag after it was chosen.
(i) Define the sample space S and determine $|S|$.
(ii) Determine the probability that I choose two silver coins and three gold coins in any order, explaining the steps in your calculation.

(b) The experiment in (a) is repeated but the five coins are chosen without replacement.
(i) Define the sample space T and determine $|T|$.
(ii) Determine the probability that I choose two silver coins and three gold coins in any order, explaining the steps in your calculation.

(c) The experiment in (a) is repeated but the five coins are chosen simultaneously.
(i) Define the sample space U and determine $|U|$.
(ii) Determine the probability that I choose two silver coins and three gold coins, explaining the steps in your calculation.

(d) Explain why your answers in (b)(ii) and (c)(ii) are the same.

AQ1. In a group of 60 people 30 are supporters of the Labour Party. A pollster chooses six people at random from the group. For $i = 0, 1, \ldots, 6$, let L_i be the event "the number of Labour Party supporters among the six chosen people is i".

(a) Determine $\mathbb{P}(L_2)$.

(b) Explain why $\mathbb{P}(L_2) = \mathbb{P}(L_4)$.

AQ2. You are dealt a hand of five cards from a standard well shuffled deck of cards.[1] Find the probability that you pick up each of the following hands:

(a) A flush (all cards of the same suit)

(a) A straight (5 consecutive cards of any suit with Ace counting high or low)

(c) Four of a kind (four of the same rank and one extra card)

(d) A full house (three of one rank and two of another rank)

AQ3. Let X be a set with n elements.

(a) Prove that the number of subsets of X with even cardinality is equal to the number of subsets of X with odd cardinality. (*Hint: Let p be the number of subsets of X with even cardinality and q be the number of subsets of X with odd cardinality. Express p and q as sums of binomial coefficients and then use the binomial theorem to deduce that $p - q = 0$.*)

(b) Suppose I choose a subset of X at random. What is the probability that it has even cardinality?

[1] A deck of playing cards is made up of 52 cards split into 4 suits ($\clubsuit, \diamondsuit, \heartsuit, \spadesuit$) with each suit made up of one card of each of 13 ranks ($2, 3, 4, \ldots, 10$,Jack,Queen,King,Ace).