MTH4107 Introduction to Probability – 2010/11

Exercise Sheet 3

These questions are designed to help you understand the material covered in week 3 lectures. You should write up your solution to the starred question, Q3, **clearly** *and hand it in during your week 5 exercise class for feedback. Put your* **full name and student number** *on the top of your solution. It is important that you make a serious attempt to do* **all** *of questions Q1-Q3 before week 5 lectures begin. Questions AQ1-AQ2 are for additional practice. You should attempt them when you have time. Q1 recalls the notion of events as subsets of the sample space from week 1. The other questions cover the axioms and basic properties of probability. In addition to your lecture notes, material relating to this can be found in Devore, Chapter 2 Section 2.2 and Ross, Chapter 2 Sections 2.3-4.*

Q1. One student is chosen at random from a set S of students. Let F be the event "the chosen student speaks French", G be the event "the chosen student speaks German", and H be the event "the chosen student speaks Hungarian".

(a) Express the following events using set theoretic notation:

(i) the chosen student speaks both German and Hungarian;

(ii) the chosen student speaks French and German but not Hungarian;

(iii) the chosen student speaks exactly two of the languages.

(b) Express the following events in words referring specifically to what they mean for the chosen student:

(i) H^c;

(ii) $F^c \cap G^c \cap H^c$;

(iii) $(F \cup G \cup H)^c$.

(c) You are told that there are 100 students in total and that exactly 30 of them speak German. You are also told that no student speaks both French and German and that all Hungarian speakers also speak German. Express each of these statements using set theoretic notation.

Q2. Let A and B be events with $\mathbb{P}(A) = 2/5$, $\mathbb{P}(B) = 1/2$, $\mathbb{P}(A \cap B) = 3/20$. Calculate the following probabilities.

(a) $\mathbb{P}(A^c)$

(b) $\mathbb{P}(A \cup B)$

(c) $\mathbb{P}(A^c \cap B)$

(d) $\mathbb{P}(B \setminus A)$

Q3*

(a) Of the books in a public library 30% are non-fiction books, 70% are hardback books, and 20% are hardback non-fiction books. A book is chosen at random from the library with all choices equally likely. Determine the probabilities that it is:

(i) a fiction book;

(ii) either a hardback or non-fiction book;

(iii) a hardback fiction book.

(b) Prove that if A and B are events in a sample space then:

(i) $\mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1$;

(ii) $\mathbb{P}(A \triangle B) = \mathbb{P}(A) + \mathbb{P}(B) - 2\mathbb{P}(A \cap B)$.

Justify each step in your proofs by referring to an axiom or result from the on-line notes.

AQ1. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Define a probability for S by putting

$$\mathbb{P}(A) = \frac{1}{12}\left(|A \cap \{1, 2, 3, 4\}| + 2|A \cap \{5, 6, 7, 8\}|\right).$$

for each $A \subseteq S$. Verify that this definition satisfies Kolmogorov's axioms for probability.

AQ2. Let A, B, C be events.

(a) Prove that

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B \setminus A) + \mathbb{P}(C \setminus (A \cup B)).$$

(b) Deduce that

$$\mathbb{P}(A \cup B \cup C) \leq \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C).$$