MTH4107 Introduction to Probability – 2010/11

Exercise Sheet 2

These questions are designed to help you understand the material covered in week 2 lectures. You should write up your solution to the starred question, Q3, **clearly** *and hand it in during your week 4 exercise class for feedback. Put your* **full name and student number** *on the top of your solution. It is important that you make a serious attempt to do* **all** *of questions Q1-Q3 before week 4 lectures begin. Questions AQ1-AQ3 are for additional practice. You should attempt them when you have time.*

Q1. In each of the following cases say whether or not f defines a function from A to B. Give brief explanations.

(a) $A = \{1, 2, 3, 4\}, \quad B = \{1, 2, 3, 4\}, \quad f(x) = x - 1.$

(b) $A = \{1, 2, 3, 4\}, \quad B = \{1, 2, 3, 4\}, \quad f(x) = 5 - x.$

(c) $A = B = \mathbb{R}$,

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$$

(d) $A = B = \mathbb{R}$,

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x \geq 1 \end{cases}$$

Q2. For each of the following functions f say whether or not f is surjective, whether or not f is injective, and whether or not f is bijective. Give brief explanations. For those functions which have an inverse say what the inverse function is. (You are asked lots of things about each function in this question; make sure you don't miss any of them out!)

(a) $f : \mathbb{N} \to \mathbb{N}, \quad f : x \mapsto 3x + 4$

(b) $f : \mathbb{R} \to \mathbb{R}, \quad f : x \mapsto 3x + 4$

(c) $f : \mathbb{R} \to \{y \in \mathbb{R} : -1 \leq y \leq 1\}, \quad f : x \mapsto \sin(x)$

(d) $f : \mathbb{R} \to \mathbb{R}, \quad f : x \mapsto \begin{cases} -x^2 & \text{if } x \leq 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

Q3*. Let $f : \mathbb{N} \to \mathbb{Z}$ be defined by

$$f(x) = \begin{cases} x/2 & \text{if } x \in \mathbb{N} \text{ is even,} \\ -(x+1)/2 & \text{if } x \in \mathbb{N} \text{ is odd.} \end{cases}$$

(a) Show that f is a bijection.
(b) Find an inverse function $g : \mathbb{Z} \to \mathbb{N}$. Verify that your function g is indeed an inverse to f.

AQ1. For each of the following either give an example of such a function or say why there is no such function:

(a) an injective function $f : \{1, 2, 3, 4\} \to \{1, 2, 3\}$,

(b) a surjective function $f : \{1, 2, 3, 4\} \to \{1, 2, 3\}$,

(c) an injective function $f : \{1, 2, 3\} \to \{1, 2, 3, 4\}$,

(d) a surjective function $f : \{1, 2, 3\} \to \{1, 2, 3, 4\}$.

AQ2.

(a) Find a function $f : \mathbb{N} \to \mathbb{N}$ which is injective but not surjective.

(b) Find a function $f : \mathbb{N} \to \mathbb{N}$ which is surjective but not injective.

(c) Let A be a finite set. Is it possible to find a function $f : A \to A$ which is injective but not surjective, or surjective but not injective? Why?

AQ3. Let A, B, C be finite sets. Show that if $f : A \to B$ and $g : B \to C$ are both bijections then $g \circ f : A \to C$ is a bijection.