MTH4107 Introduction to Probability – 2010/11

Solutions to Exercise Sheet 1

Q1.

(i) If the ordered pair (i, j) denotes that the first roll is number i and the second roll is number j then the sample space is

$$\{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\}.$$

You could also write this (using the notation for Cartesian product) as

$$\{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}.$$

or you could write it out explicitly as

$$\begin{gathered}\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}.\end{gathered}$$

(ii) The sample space has 36 elements. (Either just count them or note that there are 6 possibilities for the first roll and for each of these there are 6 possibilities for the second. Hence, there are $6 \times 6 = 36$ possibilities in total).

Q2.

(i)
$$S = \{hh, hth, htth, httt, thh, thth, thtt, tthh, ttht, ttt\}$$

(ii) This event is
$$\{htth, httt, thth, thtt, tthh, ttht\}$$

(iii) Two elements of the sample space are $ttthh$ and $htttttttttth$. Of course there are many other choices. Any sequence which ends with a tail and contains at least two heads and exactly three tails will do, as will any sequence which ends with a head and contains at least three tails and exactly two heads.

In contrast to part (i) this sample space is infinite. For instance, there are infinitely many sequences of the form k heads followed by 3 tails in S. Another difference is the sample space contains some infinite sequences.

Q3.

(i) The simplest way to express S is to list the horses which finish in the order in which they finish (so CA means C finishes first, A finishes second, B falls). We record the outcome that all horses fall as X. You may have chosen a different notation which is fine provided that you explained it. The sample space is

$$\{X, A, B, C, AB, BA, AC, CA, BC, CB, ABC, ACB, BAC, BCA, CAB, CBA\}.$$

(ii) With the notation above this event is

$$\{A, AB, AC, ABC, ACB\}.$$

(iii) With the notation above this event is

$$\{X, A, C, AC, CA\}.$$

(iv) With the notation above this event is

$$\{ABC, ACB, BAC, BCA, CAB, CBA\}.$$

Q4.

(i) $A \cap B = \{3, 4\}$, $|A \cap B| = 2$.

(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $|A \cup C| = 8$.

(iii) $B \triangle C = \{3, 4, 7, 8\}$, $|B \triangle C| = 4$.

Q5*

(a) A sample space is the set of all possible outcomes on an experiment. An event is a subset of the sample space.

(b) (i)

$$\{(x, y, z) \ : \ x, y, z \in \{1, 2, 3, 4, 5\} \text{ and } x \neq y \neq z \neq x\}.$$

The sample space has 60 elements. (Either just count them or note that there are 5 possibilities for the first card, 4 possibilities for the second and 3 possibilities for the third. Hence, there are $5 \times 4 \times 3 = 60$ possibilities in total.)

(ii) $\{(2, y, 4) \ : \ y \in \{1, 3, 5\}\}$.

(iii) $(T \cup F^c)^c = T^c \cap (F^c)^c = T^c \cap F$ by De Morgan's law. So it is the event that 4 is chosen and 2 is not chosen.

Please let me know if you have any comments or corrections