MTH4107 Introduction to Probability – 2010/11

Exercise Sheet 10

These questions are designed to help you understand cumulative distribution functions and continuous random variables. You should discuss questions Q1-Q2 in your week 12 exercise classes. Question AQ1 is for additional practice. You should attempt it when you have time.
In addition to your lecture notes, material relating to these questions can be found in Devore, Chapter 4 Sections 4.1-2. or Ross, Chapter 5 Sections 5.1-3.

Q1.

(a) A discrete random variable A has the following probability mass function:

n	-1	0	1	2
$P(A=n)$	$1/10$	$3/5$	$1/5$	$1/10$

Find the cumulative distribution function of A.

(b) A discrete random variable B has cumulative distribution function:

$$F_B(r) = \begin{cases} 0 & \text{if } r < 1/2 \\ 1/5 & \text{if } 1/2 \leq r < 3 \\ 3/5 & \text{if } 3 \leq r < 5 \\ 1 & \text{if } 5 \leq r \end{cases}$$

Write down the range of B. Find the probability mass function of B.

Q2. A continuous random variable X has cumulative distribution function:

$$F_X(r) = \begin{cases} 0 & \text{if } r \leq 0 \\ \sqrt{r} & \text{if } 0 < r \leq 1 \\ 1 & \text{if } r > 1 \end{cases}$$

(a) Find the probability density function of X.

(b) Calculate the expectation and variance of X.

(c) Calculate the lower quartile of X.

AQ1. A continuous random variable Y has probability density function:

$$f_Y(r) = \begin{cases} 0 & \text{if } r < 1 \\ (r-1) & \text{if } 1 \leq r < 2 \\ (3-r) & \text{if } 2 \leq r < 3 \\ 0 & \text{if } r \geq 3 \end{cases}$$

(a) Find the cumulative distribution function of Y.

(b) Use the properties of a cumulative distribution function to write down a couple of ways in which you could check that your answer to part (a) is plausible. Perform those checks and revisit part (a) if appropriate.

(c) Calculate $\mathbb{P}(3/2 < Y < 5/2)$ by evaluating the cumulative distribution function at appropriate places.

(d) Calculate $\mathbb{P}(3/2 < Y < 5/2)$ by integrating the probability density function and check you get the same answer.