# MTH4107 Introduction to Probability

## End-term Test

10:00am, Friday 17 December 2010

Duration: 40 minutes

*Answer All Questions. Each question carries 25 marks.*

*Calculators are NOT permitted in this examination.*

*Write your answers on the question paper. If you need more space then use the blank pages at the end of the booklet and be sure to number your answers clearly.*

SURNAME:

FIRST NAME(S):

STUDENT NUMBER:

EXERCISE CLASS GROUP:

**1.**

(a) State Bayes' Theorem (you are not required to prove it).

(b) A certain disease is carried by 1% of a population. If a person with the disease is tested for it there is probability 9/10 that the test shows that they have the disease. If a person without the disease is tested for it there is probability 1/9 that the test shows they have the disease. A randomly chosen person is tested for the disease.

(i) Calculate the probability that the test shows they have the disease.

(ii) Calculate the conditional probability that they have the disease given that the test shows that they do have it.

**2.**

(a) Define the *expected value* and *variance* of a discrete random variable $X$.

(b) A bag contains three balls labeled 0, 1 and 2. I select two balls at random, one after the other and with replacement. Let $X$ be the maximum label on the two selected balls.

(i) Determine the probability mass function of $X$.

(ii) Calculate the expected value and variance of $X$.

**3.** Let $\lambda$ be a positive real number, and $X$ be a discrete random variable which has probability mass function $P(X = m) = e^{-\lambda}\lambda^m/m!$ for each integer $m \geq 0$.

(a) Give the commonly used name for the probability distribution of $X$ and describe a practical situation in which it occurs.

(b) Prove that $\mathrm{E}(X) = \lambda$.

**4.** Two discrete random variables $X$ and $Y$ defined on the same sample space have the following joint probability mass function.

| | | $X$ | | |
|---|---|---|---|---|
| | | 0 | 1 | 2 |
| | 0 | 1/9 | 1/6 | 0 |
| $Y$ | 1 | 1/6 | 1/9 | 1/6 |
| | 2 | 0 | 1/6 | 1/9 |

(a) Construct the probability mass functions for each of $X$ and $Y$.

(b) Calculate $\mathrm{E}(X)$, $\mathrm{E}(Y)$, $\mathrm{E}(XY)$ and $\mathrm{Cov}(X, Y)$.

(c) Are $X$ and $Y$ independent? Justify your answer.

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