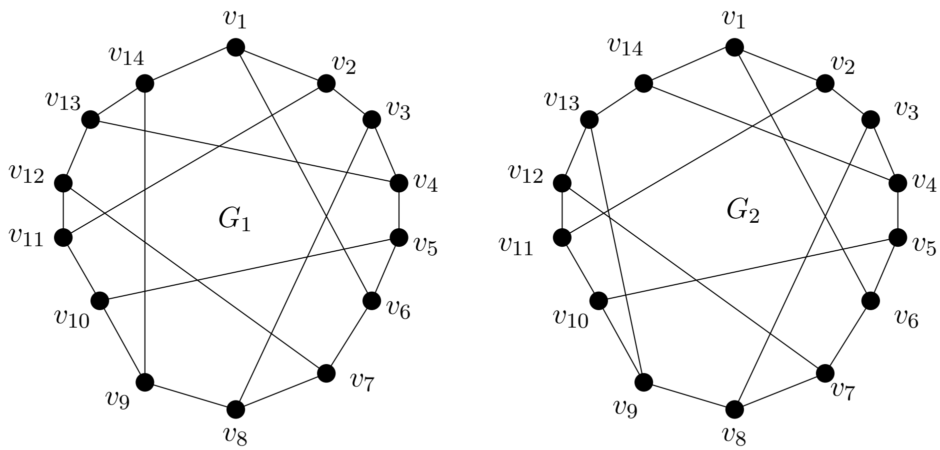
MAS210 Graph Theory Exercises 7

Hand in to BLUE BOX on the GROUND FLOOR of math sci building before 4:30pm on Friday 16/3/07.

Q1 Determine whether each of the following graphs G_1 and G_2 are bipartite. Justify your answers.



[20]

Q2 (a) Prove that if a graph G contains a cycle of odd length then G is not bipartite.

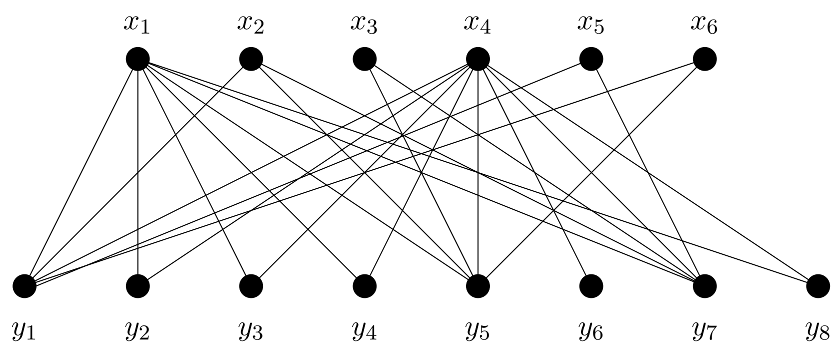
(b) Suppose G is a connected graph which contains no cycles of odd length. Choose $v_0 \in V(G)$ and let T be a spanning tree of G rooted at v_0. Let $X = \{v \in V(G) : dist_T(v_0, v) \text{ is even}\}$ and $Y = \{v \in V(G) : dist_T(v_0, v) \text{ is odd}\}$. Prove that G is bipartite with bipartition $\{X, Y\}$.

(c) Deduce that a graph is bipartite if and only if it contains no cycles of odd length.

[20]

TURNOVER

Q3 Use König's algorithm to construct a maximum matching and a minimum cover in the following bipartite graph, starting with the matching $M_1 = \{x_1y_1, x_2y_5, x_3y_7, x_4y_2\}$.



Justify the facts that your matching is maximum and your cover is minimum. [40]

Q4 Use König's theorem to construct a connected bipartite graph G with bipartition $\{X, Y\}$ such that $|X| = 6 = |Y|$, $match(G) = 4$, and $d_G(v) \geq 2$ for all $v \in V(G)$. Justify the fact that your graph G has $match(G) = 4$. [20]