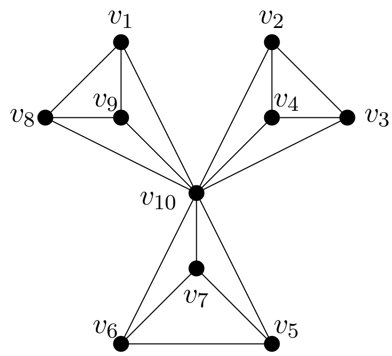
**MAS210 Graph Theory Exercises 6**

*Hand in to BLUE BOX on the GROUND FLOOR of math sci building before 4:30pm on Friday 9/3/07.*

Q1 Consider the following graph $G$.




Find a maximum matching in $G$ and a minimum cover of $G$. Justify the facts that your matching is maximum and your cover is minimum. [30]

Q2 The *complete graph* $K_n$ is the graph with $n$ vertices in which each vertex is joined to every other vertex by an edge. The *complete bipartite graph* $K_{m,n}$ is the graph with vertices partitioned into two sets $X, Y$ where $|X| = m$, $|Y| = n$, and in which each vertex of $X$ is joined to every vertex of $Y$ by an edge.

(a) Draw $K_4$, $K_5$, $K_{3,3}$, and $K_{3,4}$.

(b) Determine $match(K_n)$, $cov(K_n)$, $match(K_{m,n})$, and $cov(K_{m,n})$ for all integers $1 \leq m \leq n$. Justify your answers. [40]

Q3 (a) Write down the iterative step in Dijkstra's algorithm.

(b) Prove that the time taken by the $(i+1)$'th iteration of Dijkstra's algorithm applied to a network $N$ is $O(|V(N)| + d_N(x_{i+1}))$, assuming that all elementary arithmetic operations can be performed in constant time no matter how large the numbers involved are.

(c) Deduce that, under the same assumption, the total time taken when Dijkstra's algorithm is run on $N$ is $O(|V(N)|^2 + |(E(N)|)$. Hence, deduce that Dijkstra's algorithm is strongly polynomial. [30]