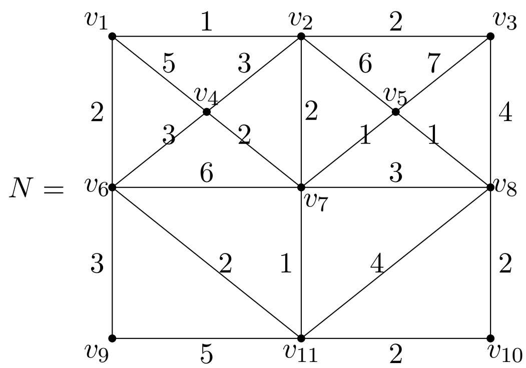
MAS210 Graph Theory Exercises 4

Hand in to BLUE BOX on the GROUND FLOOR of math sci building before 4:30pm on Friday 9/2/07.

Q1 Consider the following network N.

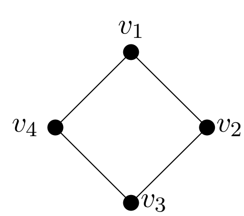


(a) An implementation of Dijkstra's algorithm starting at v_1 produces the following tree T_4 at the end of the fourth iteration: $V(T_4) = \{v_1, v_2, v_3, v_6\}$ and $E(T_4) = \{v_1v_2, v_2v_3, v_1v_6\}$. It also gives the vertex labels shown in the following table.

v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}
x_1	x_2	x_4	$[x_2, 4]$	$[x_2, 7]$	x_3	$[x_2, 3]$	$[x_4, 7]$	$[x_3, 5]$	$[x_1, \infty]$	$[x_3, 4]$

List the edge(s) of N which could be added to T_4 in the next iteration and, for each such edge, give a table showing the new vertex labels. [20]

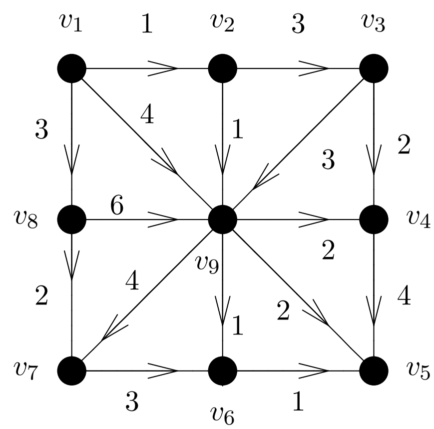
Q2 Show that Dijkstra's algorithm for constructing a shortest path spanning tree rooted at a vertex v_1 in a network may not work if the network is allowed to have edges with negative weights by assigning suitable weights to the edges in the following network.



[20]

TURNOVER

Q3 Consider the following acyclic directed network N.



(a) Construct an acyclic labeling of the vertices of N.
(b) Use your acyclic labeling to find a spanning out-arborescence of N rooted at v which contains longest directed paths from v_1 to every vertex of N.
(c) Use your acyclic labeling to find a spanning out-arborescence of N rooted at v which contains shortest directed paths from v_1 to every vertex of N.

[30]

Q4 Let N be an acyclic directed network and v be a vertex of N such that N contains a directed path from v to every vertex of N. Let T_i be an out-arborescence rooted at v produced in the i'th iteration of Morávek's algorithm for finding longest paths applied to N.
(a) Prove that the unique path in T_i from v to each vertex x of T_i is a longest path in N from v to x.
(b) Does your proof assume that N has positive edge weights? [30]