Time Series Course-work 6
Solutions to the Theory Questions

Question 6.2.1

(a) Mixed seasonal $ARMA(1,0) \times (0,1)_{12}$

(b) Seasonal $MA(2)_4$

(c) $AR(1)$

(d) Seasonal $ARIMA(2,0,1) \times (0,1,0)_{12}$

(e) $ARIMA(2,1,0)$

Question 6.2.2

An $AR(1)_4$ model can be written as $(1 - \Phi B^4)X_t = Z_t$ or $\Phi(B^4)X_t = Z_t$. To obtain the ACF it is convenient to represent is as a linear process of the form

$$X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j} = \psi(B)Z_t,$$

where $\psi(B) = \sum_{j=0}^{\infty} \psi_j B^j$. Then we can write

$$X_t = \psi(B)Z_t = \psi(B)\Phi(B^4)X_t,$$

which means that

$$1 = \psi(B)\Phi(B^4)$$

or in full

$$1 = (1 + \psi_1 B + \psi_2 B^2 + \psi_3 B^3 + \psi_4 B^4 + \psi_5 B^5 + ...)(1 - \Phi B^4).$$

The right hand side can be rearranged to

$$1+\psi_1 B+\psi_2 B^2+\psi_3 B^3+(\psi_4-\Phi)B^4+(\psi_5-\psi_1\Phi)B^5+(\psi_6-\psi_2\Phi)B^6+(\psi_7-\psi_3\Phi)B^7+(\psi_8-\psi_4\Phi)B^8+...$$

Comparing the coefficients of B^j on the LHS and the RHS we obtain

$$\begin{aligned}
&\psi_0 = 1\\
&\psi_1 = \psi_2 = \psi_3 = 0\\
&\psi_4 = \Phi\\
&\psi_5 = \psi_1\Phi = 0\\
&\psi_6 = \psi_2\Phi = 0\\
&\psi_7 = \psi_3\Phi = 0\\
&\psi_8 = \psi_4\Phi = \Phi^2\\
&\quad \cdots
\end{aligned}$$

That is

$$\psi_j = \begin{cases} 0 & \text{for } j \neq 4k, k = 1, 2, ... \\ \Phi^k & \text{for } j = 4k, k = 0, 1, 2, ... \end{cases}$$

X_t is a zero mean process and by Corrolary 4.1 we have

$$\gamma(\tau) = \sigma^2 \sum_{j=0}^{\infty} \psi_j \psi_{j+\tau},$$

which can be written as

$$\gamma(\tau) = \begin{cases} \sigma^2 \sum_{j=0}^{\infty} \psi_{4j}\psi_{4j+\tau} = \sigma^2 \sum_{j=0}^{\infty} \Phi^{4j}\Phi^{4j+\tau}, & \text{for } \tau = 4k, k = 0, 1, 2, .. \\ 0, & \text{otherwise} \end{cases}$$

Hence, for $\tau = 4k, k = 0, 1, 2, ..$, we obtain

$$\gamma(\tau) = \sigma^2 \Phi^\tau \sum_{j=0}^{\infty} \Phi^{8j} = \sigma^2 \frac{\Phi^\tau}{1 - \Phi^8}.$$

Then, dividing $\gamma(\tau)$ by $\gamma(0)$ we obtain the required result.