Part 2 *Choose a correct answer for each of the following problems.*

1. Seasonality of a time series $\{X_t\}_{t=1,2,\ldots}$ means that
   (a) $\mathrm{E}(X_t)$ is an increasing function of t.
   (b) there is a repeating effect of each season over all periods in the series.
   (c) all seasonality effects are equal and the series is stationary.
   (d) the series X_t is deterministic.

2. Assume that a time series model is $X_t = m_t + Y_t$, where the trend m_t is a polynomial of degree 2 such that $m_t = 1 + 3t + 4t^2$ and Y_t denotes a zero mean random noise variable. Then the expectation of $\nabla^2 X_t$, where ∇ is the differencing operator, is
   (a) $\mathrm{E}(\nabla^2 X_t) = 8 + \nabla^2 Y_t$.
   (b) $\mathrm{E}(\nabla^2 X_t) = 4 + \nabla^2 Y_t$.
   (c) $\mathrm{E}(\nabla^2 X_t) = 8$.
   (d) $\mathrm{E}(\nabla^2 X_t) = 4$.

3. The following is true for a strictly stationary process:
   (a) all n-tuples of variables $(X_{j_1}, \ldots, X_{j_n})$ are equal.
   (b) it has a constant mean and all its autocorrelations depend on time.
   (c) it has a constant mean and a constant variance and the covariance $\mathrm{cov}(X_t, X_{t+\tau})$ depends on time for all $\tau = 1, 2....$
   (d) all variables X_t are identically distributed.

4. A real valued function defined on the integers is the autocovariance function of a stationary time series if and only if
   (a) It is an even and non-negative function of time t.
   (b) It is an even and non-negative definite function.
   (c) It is an even and non-negative function of lag τ.
   (d) It is a monotonic function of time t.

5. The time series model $X_t - 0.7X_{t-1} = Z_t + 0.5Z_{t-1}$, where $Z_t \sim WN(0, 1)$, represents
   (a) a causal AR(2) process.
   (b) an invertible MA(2) process.
   (c) a causal and invertible ARMA(1,1) process.
   (d) a noncausal but invertible ARMA(1,1) process.

6. The Autocorrelation Function of the process $X_t = Z_t + 0.5Z_{t-1}$, where $Z_t \sim WN(0, \sigma^2)$, at lag $\tau = 1$ is equal to
   (a) $\rho(1) = 0.4$.
   (b) $\rho(1) = 0.5$.
   (c) $\rho(1) = 0.25$.
   (d) $\rho(1) = -0.25$.

7. The ACF of MA(q) of the form $\frac{1}{q+1}\sum_{k=0}^{q} Z_{t-k}$, where Z_t is a White Noise random variable, is

   (a) $\rho(\tau) = \begin{cases} \frac{q-\tau}{q+1} & \text{for} \quad \tau = 0, 1, ..., q \\ 0 & \text{for} \quad \tau > q \end{cases}$

   (b) $\rho(\tau) = \begin{cases} \frac{q-1+\tau}{q+1} & \text{for} \quad \tau = 0, 1, ..., q \\ 0 & \text{for} \quad \tau > q \end{cases}$

   (c) $\rho(\tau) = \begin{cases} \frac{q+1-\tau}{q+1} & \text{for} \quad \tau = 0, 1, ..., q \\ 0 & \text{for} \quad \tau > q \end{cases}$

   (d) $\rho(\tau) = \begin{cases} \frac{\tau}{q+1} & \text{for} \quad \tau = 0, 1, ..., q \\ 0 & \text{for} \quad \tau > q \end{cases}$

8. An random walk process $X_t = X_{t-1} + Z_t$, where $Z_t \sim WN(0, \sigma^2)$,

   (a) is causal.

   (b) is stationary.

   (c) is not stationary.

   (d) has constant mean and constant variance.

9. The ACF of the process $X_t - 0.7X_{t-1} = Z_t + 0.5Z_{t-1}$ at lag 1 is $\rho(1) = 0.83$. The ACF at lag two and at lag three is, respectively, equal to:

   (a) $\rho(2) = 0.25$, $\rho(3) = 0.125$.

   (b) $\rho(2) = 0.49$, $\rho(3) = 0.343$.

   (c) $\rho(2) = 0.581$, $\rho(3) = 0.4067$.

   (d) $\rho(2) = 0.415$, $\rho(3) = 0.2075$.

10. The confidence bounds estimates for $\rho(\tau)$ of an IID process are useful for

   (a) determining a type of a stationary time series model for the given data.

   (b) testing the null hypothesis that the model parameters are non-significant.

   (c) testing the null hypothesis that the autocorrelations of the residuals are between -1 and 1.

   (d) determining if the data are a realization of a stationary process.