

**NOTE:**

*The test consists of two parts: a practical Minitab output for a set of data and a set of theory questions. You should attempt all questions in both parts.*

*It is a multiple choice test with 20 problems altogether. Choose only one statement for each problem, which you think is true, and mark it on the answer sheet by crossing a box.*

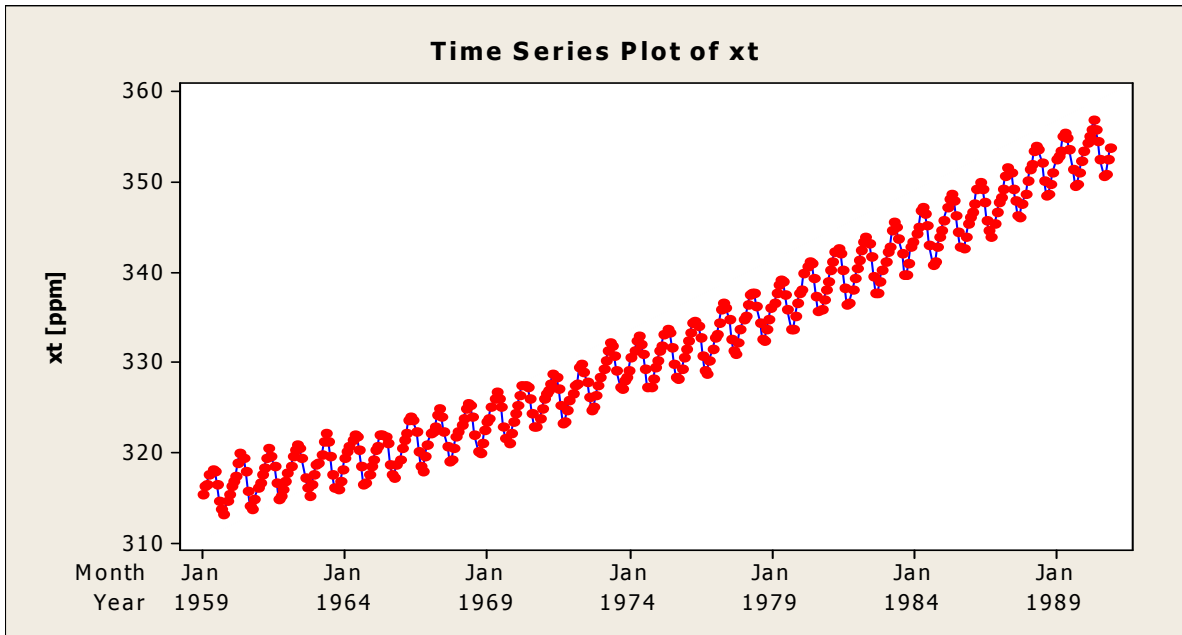
*For each correct answer you get 1 mark, for no answer you get 0 and for a wrong answer you get minus 0.25. The total is then scaled to 0 – 100 range.*

*Total time for the test is 40 minutes. Calculators are not permitted in this test.*

You must not start to read the questions  
until instructed to do so by the invigilator.

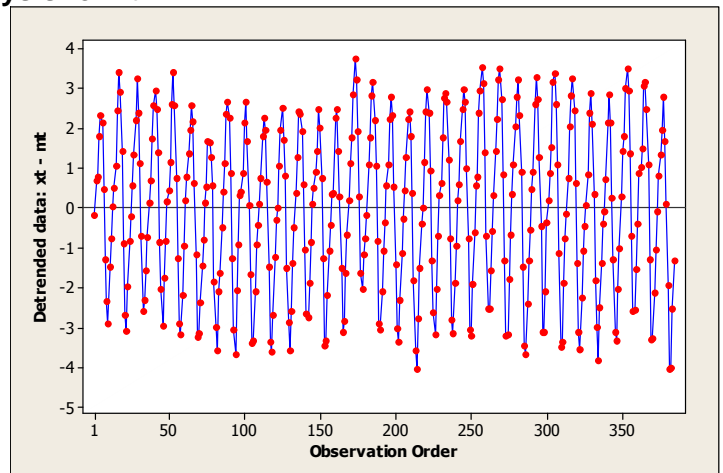
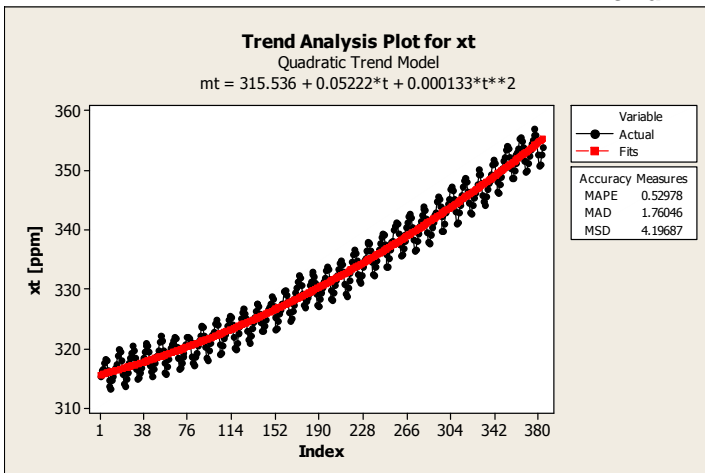
# Part 1

Below there is a Minitab output with comments on a time series analysis of monthly measurements of carbon dioxide ( $X_t$ ) above Mauna Loa, Hawaii, Jan 1959 - Dec 1990. Units: parts per million (ppm). Choose the right comment.



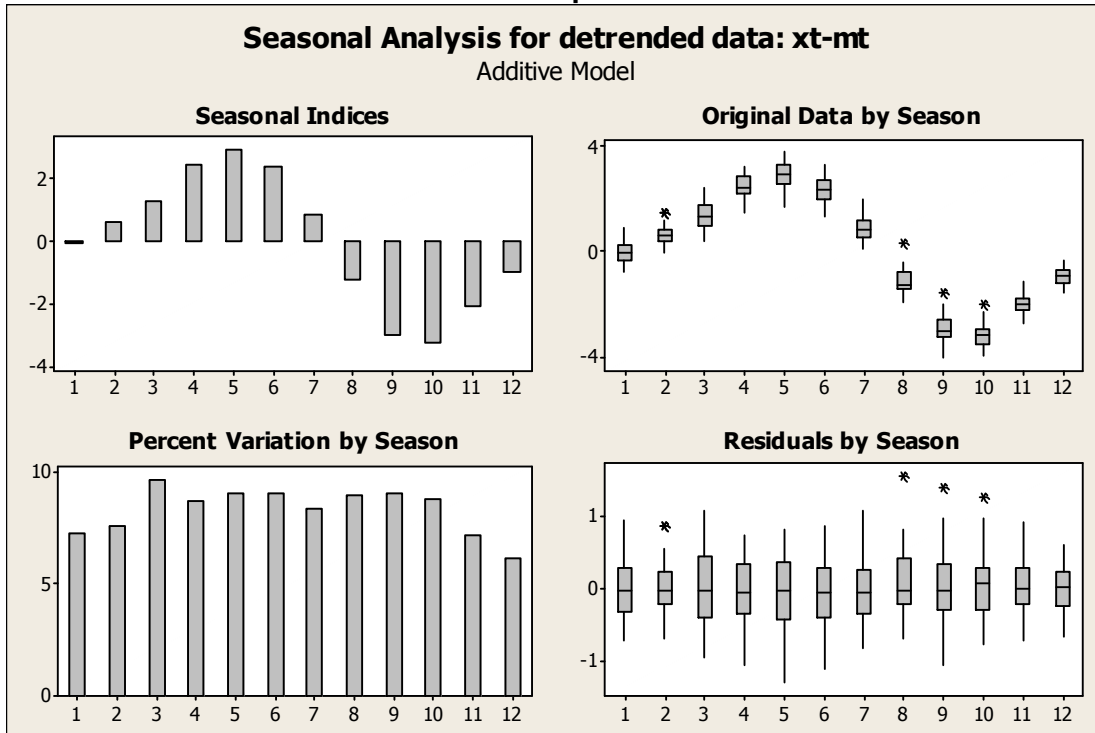
- 1 The time series plot indicates that:
  - (a) There is a clear seasonality and also an increasing trend in the data.
  - (b) There is neither trend nor seasonality, only increasing variability of the data.
  - (c) There is an increasing trend as well as increasing variability and also seasonality in the data.
  - (d) There is seasonality and increasing variability in the data, but no trend

## Trend Analysis for $x_t$



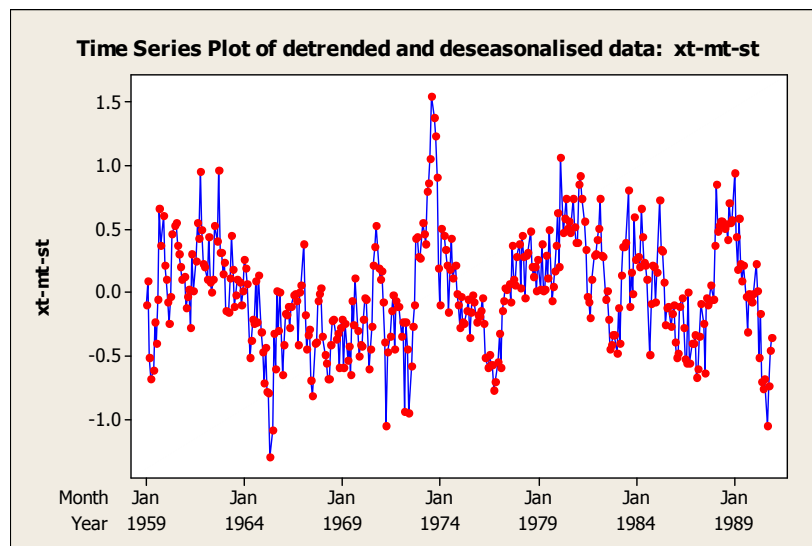
- 2 The plot of detrended data indicates that:
  - (a) The detrended data represent seasonality effects only.
  - (b) The seasonality effects are successfully removed from the data.
  - (c) A linear trend function would have better fitted the data.
  - (d) The detrended data oscillate about zero, but seasonality effects and noise effects are still present.

## Time Series Decomposition for $xt-mt$



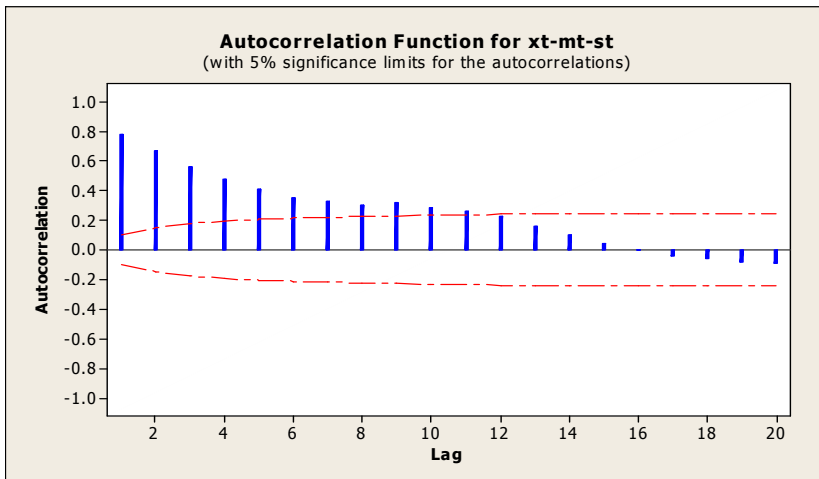
3 The plots of the analysis of seasonal effects indicate that:

- (a) On average, the carbon dioxide levels are highest in May and lowest in October.
- (b) October is the season with greatest variability in levels of carbon dioxide.
- (c) The plot 'Original Data by Season' shows that the seasonal effects are the same in each month.
- (d) The plot 'Residuals by Season' shows there is no noise in the data.



4 The time series plot of the detrended and deseasonalised data indicates that:

- (a) There is increasing variability in the detrended and deseasonalised data.
- (b) There is neither a global trend nor seasonality shown in the plot, but some local trends are present.
- (c) The seasonality effects are not successfully removed from the detrended data.
- (d) The detrended and deseasonalised data are a realization of an IID process.



Lag	ACF	T	LBQ
1	0.774106	15.17	231.91
2	0.672069	8.88	407.17
3	0.558764	6.22	528.64
4	0.476878	4.84	617.34
5	0.408946	3.92	682.75
6	0.354856	3.27	732.12
7	0.322205	2.89	772.94
8	0.303689	2.67	809.30
9	0.320679	2.77	849.94
10	0.284275	2.40	881.97

5 The sample ACF of the detrended and deseasonalised data indicates that:

- (a) The correlations have a tailing off pattern.
- (b) The correlations cut off after lag 15.
- (c) All correlations are non-significantly different from zero.
- (d) The correlations suggest a White Noise model for the detrended and deseasonalised data.

**ARIMA Model: xt-mt-st**

Final Estimates of Parameters

Type	Coef	SE Coef	T	P
AR 1	0.8618	0.0331	26.06	0.000
MA 1	0.2203	0.0633	3.48	0.001

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	14.6	22.2	37.3	51.5
DF	10	22	34	46
P-Value	0.147	0.451	0.320	0.267

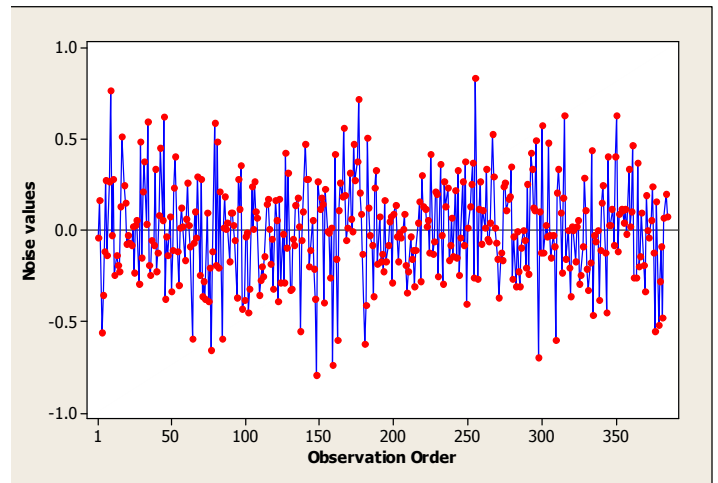
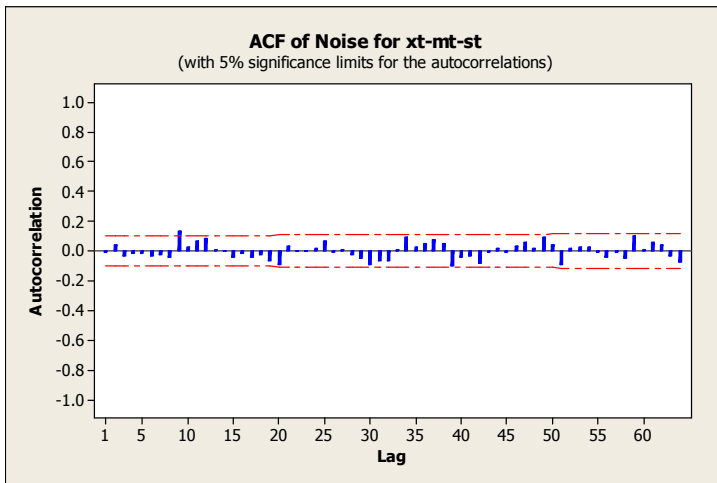
6 The numerical output given above (for the detrended and deseasonalised data  $y_t$ ) shows that the fitted model is

- (a)  $y_t - 0.2203 y_{t-1} = z_t + 0.8618 z_{t-1}$ ,
- (b)  $y_t - 0.2203 y_{t-1} = y_t - 0.8618 z_{t-1}$ ,
- (c)  $y_t - 0.8618 y_{t-1} = z_t + 0.2203 z_{t-1}$ ,
- (d)  $y_t - 0.8618 y_{t-1} = z_t - 0.2203 z_{t-1}$ ,

where  $z_t$  is a realization of  $WN(0, \sigma^2)$ .

7 The Ljung-Box Chi-square statistic shown in the numerical output of the 'ARIMA Model' indicates that

- (a) The groups of autocorrelations of the detrended and deseasonalised variables for all lags up to 12, 24, 36 and 48 are non-significant.
- (b) The autocorrelations of the detrended and deseasonalised variables which are exactly 12, 24, 36 and 48 lags apart are non-significant.
- (c) The autocorrelations of the model noise variables which are exactly 12, 24, 36 and 48 lags apart are non-significant.
- (d) The groups of autocorrelations of the model noise variables for all lags up to 12, 24, 36 and 48 are non-significant.



- 8 The graphs given above suggest that the noise values are a realization of:
- (a) An uncorrelated random process with zero mean and possibly a constant variance.
  - (b) An identically, independently distributed random variable.
  - (c) An MA(1) process with zero mean.
  - (d) A strictly stationary process.

**ARIMA Model: xt-mt-st**

Type	Coef	SE Coef	T	P
AR 1	0.6357	0.0512	12.41	0.000
AR 2	0.1933	0.0599	3.23	0.001
AR 3	-0.0143	0.0514	-0.28	0.782

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	12.9	20.0	33.8	47.6
DF	9	21	33	45
P-Value	0.169	0.519	0.431	0.369

- 9 The numerical output, given above, for another model of the detrended and deseasonalised data, shows that (when compared to the previous model, see question 6), it is:
- (a) A better model because it is more parsimonious than the previous one.
  - (b) A worse model because the p-values in the Ljung-Box tests are larger than in the previous one.
  - (c) A worse model because it is over-parameterized, that is, there is an insignificant parameter in the model.
  - (d) A worse model because one of the AR parameters is negative.
- 10 Let  $m_t = 315.536 + 0.05222 t + 0.000133 t^2$  denote the estimated trend,  $s_t = s_{t+12}$  be the estimated seasonal effects and  $y_t$  be a stationary random process. The carbon dioxide levels above Mauna Loa, in years 1959 – 1990, can be represented as:
- (a)  $x_t = m_t s_t + y_t$
  - (b)  $x_t = m_t + s_t y_t$
  - (c)  $x_t = m_t s_t y_t$
  - (d)  $x_t = m_t + s_t + y_t$