Queen Mary
University of London
School of Mathematical Sciences

MTH6139 TIME SERIES — Sample Test 1

NOTE:

The test consists of two parts: a practical Minitab output for a set of data and a set of theory questions. You should attempt all questions in both parts.

It is a multiple choice test with 20 problems altogether. Choose only one statement for each problem, which you think is true, and mark it on the answer sheet by crossing a box.

For each correct answer you get 1 mark, for no answer you get 0 and for a wrong answer you get -0.25. The total is then scaled to 0 – 100 range.

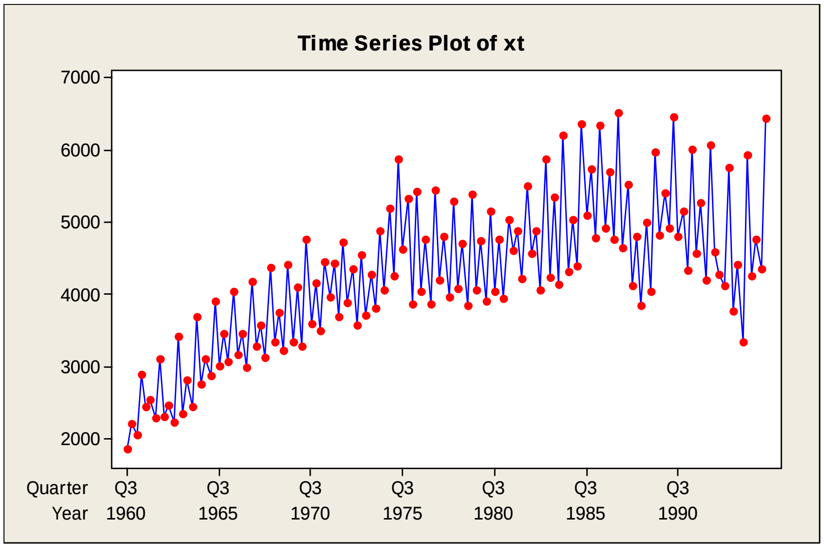
Total time for the test is 40 minutes. Calculators are not permitted in this test.

You must not start to read the questions

until instructed to do so by the invigilator.

Part 1

Below there is a Minitab output with comments of a time series analysis of quarterly data of gross fixed capital public expenditure in Australia; $million 1989/90 price, Sep 60 - Sep 1995. Choose the right comment.



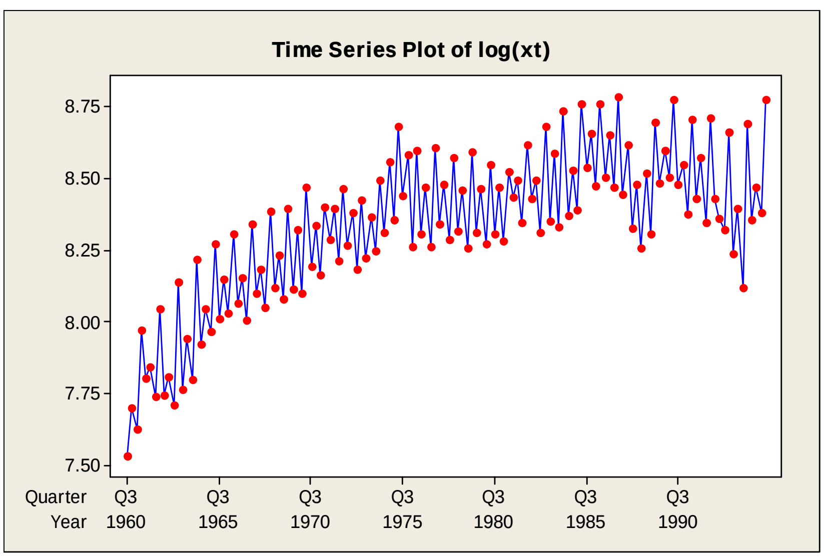
1 The time series plot indicates that

(a) There is an increasing trend and variability but no seasonality in the data.

(b) There is neither trend nor seasonality, only increasing variability of the data.

(c) There is an increasing trend as well as increasing variability and also seasonality in the data.

(d) There is seasonality and increasing variability in the data, but no trend.



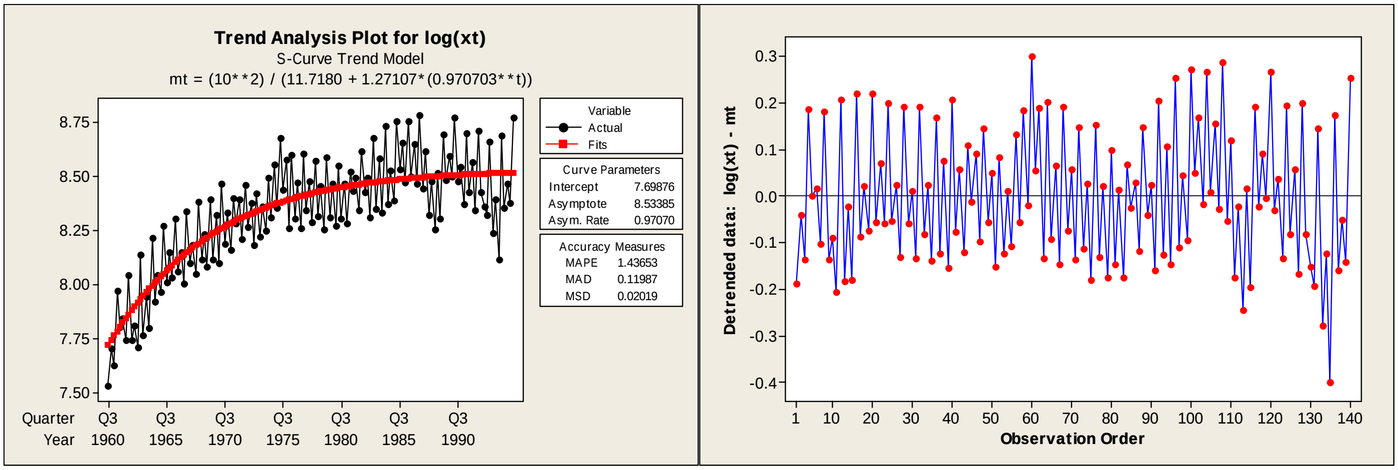
2 The plot of the log transformed data indicates that:

(a) The seasonality effects are removed from the data.

(b) The variability of the data is stabilized.

(c) The time series is now stationary.

(d) The trend in the data is less apparent.



3 The plot of detrended data indicates that:

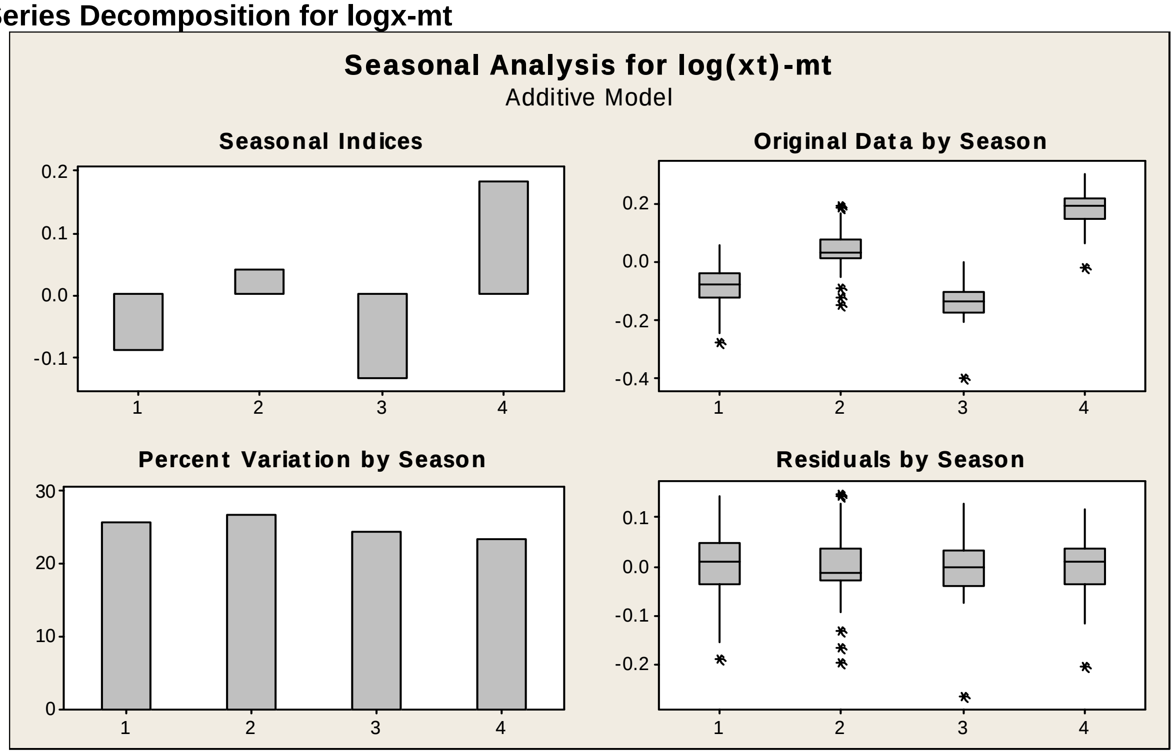
(a) The detrended data represent a stationary process.

(b) The seasonality effects are successfully removed from log(x_t).

(c) The trend function is not appropriate for these data.

(d) The detrended data oscillate about zero, but seasonality effects are still present.

Time Series Decomposition for logx-mt



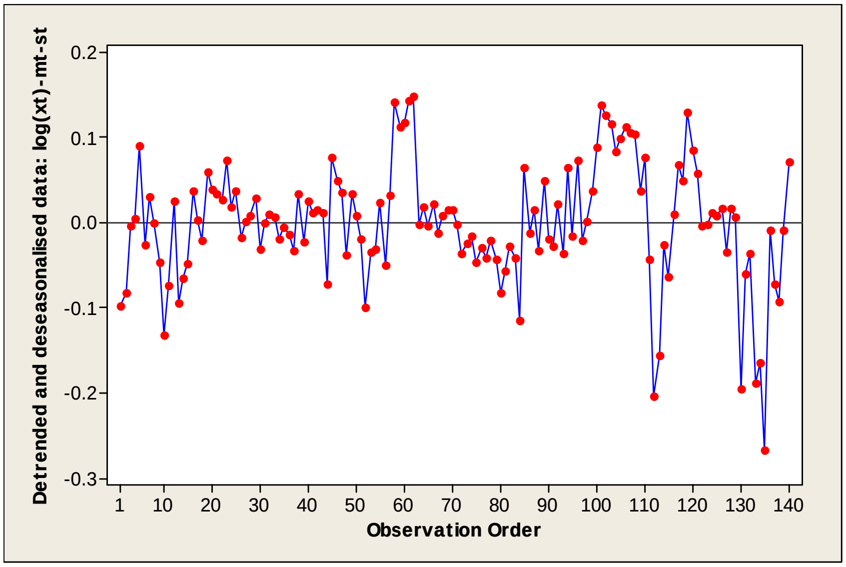
4 The plots of seasonal effects indicate that:

(a) The expenditure is lowest in the third quarter and largest in the fourth quarter of a year.

(b) The variability of expenditure is very different in each season.

(c) The plot ‘Residuals by Season’ shows differences in seasonal effects.

(d) The plot ‘Original Data by Season’ shows differences in seasonal effects.



5 The time series plot of the detrended and deseasonalised data indicates that:

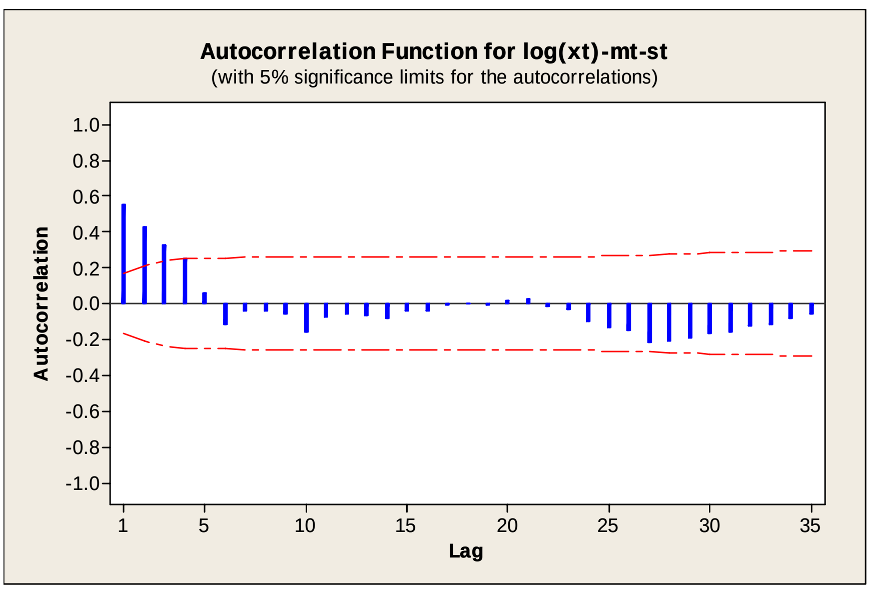
(a) There is still a clear trend in the series.

(b) The data can be treated as a realization of a stationary process.

(c) The seasonality effects are not successfully removed from the detrended data.

(d) The data can be treated as a realization of an IID process.

Autocorrelation Function for the residuals yt = log(xt) – mt – st



Lag	ACF	T
1	0.552832	6.54
2	0.429324	4.00
3	0.325060	2.73
4	0.253381	2.03
5	0.055679	0.43

6 The sample ACF of the detrended and deseasonalised data indicates that:

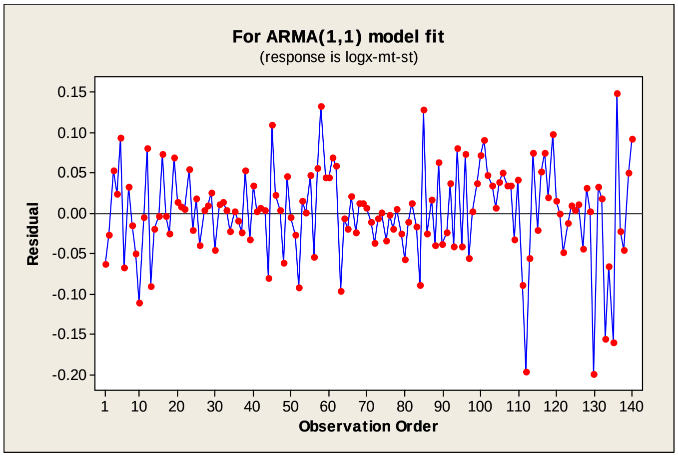
(a) There are significant correlations among the variables which are up to three or four lags apart.

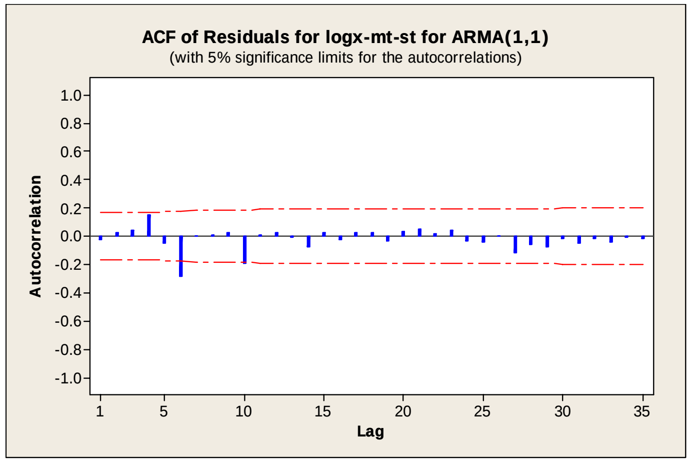
(b) The correlations depend on time.

(c) The correlations follow the pattern of a φ^{τ} function, where $\varphi < 1$ and τ denotes lag.

(d) The residuals are a realization of a White Noise variable.

Fitting ARIMA(1,0,1) model





ARIMA Model: logx – mt – st

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Final Estimates of Parameters

Type       Coef  SE Coef      T      P
AR   1   0.7555   0.0933   8.10  0.000
MA   1   0.2819   0.1374   2.05  0.042

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag             12      24      36      48
Chi-Square    22.2    24.9    30.5    42.6
DF              10      22      34      46
P-Value      0.014   0.300   0.641   0.616
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7 The given Minitab output for the fitted ARMA(1,1) model for the detrended and deseasonalised data indicates that:

(a) Both model parameters (φ and θ) are significantly different from zero.

(b) None of the groups of residuals (indicated in the Ljung-Box Q test) is significantly correlated.

(c) The residuals are a representation of an IID process.

(d) The residuals could be modeled as an AR(1) process.

8 The fitted ARMA(1,1) model is:

(a) $x_t = 0.7555x_{t-1} + 0.2819\ z_{t-1}$

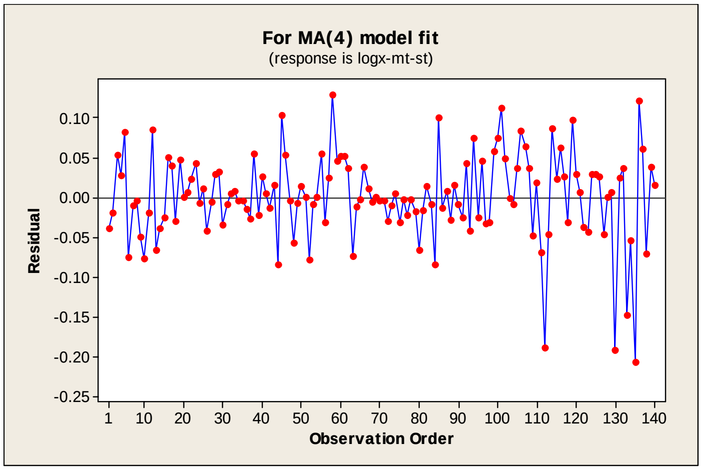
(b) $x_t - 0.7555x_{t-1} = z_t + 0.2819\ z_{t-1}$

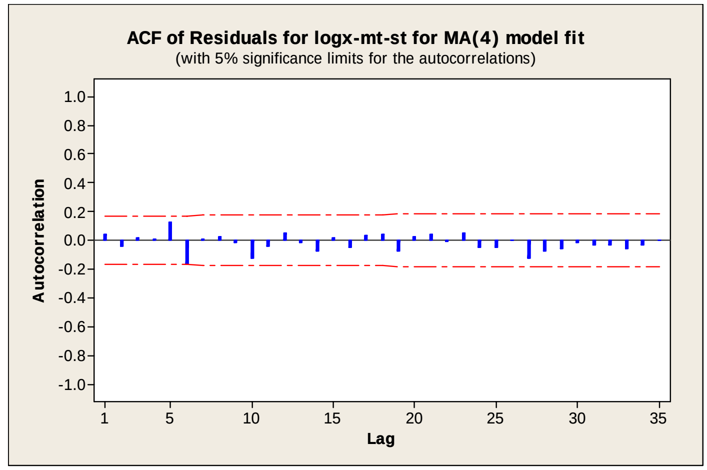
(c) $x_t - 0.7555x_{t-1} = z_t - 0.2819\ z_{t-1}$

(d) $x_t + 0.7555x_{t-1} + 0.2819\ x_{t-2} = z_t$

where z_t is the realization of a White Noise variable.

Fitting ARIMA(0,0,4) model





ARIMA Model: logx – mt – st

Final Estimates of Parameters

Type		Coef	SE Coef	T	P
MA	1	-0.3428	0.0781	-4.39	0.000
MA	2	-0.4344	0.0785	-5.53	0.000
MA	3	-0.3449	0.0789	-4.37	0.000
MA	4	-0.4367	0.0811	-5.39	0.000

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	10.1	14.1	20.1	27.5
DF	8	20	32	44
P-Value	0.261	0.827	0.949	0.975

9 The given Minitab output for the fitted MA(4) model for the detrended and deseasonalised data indicates that:

(e) None of the model parameters ($\theta_1 - \theta_4$) is significantly different from zero.

(f) None of the groups of residuals (indicated in the Ljung-Box Q test) is significantly correlated.

(g) The residuals are a representation of an IID process.

(h) The residuals could be modeled as an AR(1) process.

10 The fitted MA(4) model is:

(a) $x_t = z_t + 0.3428\, z_{t-1} + 0.4344\, z_{t-2} + 03449\, z_{t-3} + 0.4367\, z_{t-4}$

(b) $x_t + 0.3428\, x_{t-1} + 0.4344\, x_{t-2} = z_t + 03449\, z_{t-1} + 0.4367\, z_{t-2}$

(c) $x_t = z_t - 0.3428\, z_{t-1} - 0.4344\, z_{t-2} - 03449\, z_{t-3} - 0.4367\, z_{t-4}$

(d) $x_t - 0.3428\, x_{t-1} - 0.4344\, x_{t-2} = z_t + 03449\, z_{t-1} + 0.4367\, z_{t-1}$

where z_t is the realization of a White Noise variable.

Part 2 *Choose a correct answer for each of the following problems.*

1. An increasing trend of a time series $\{X_t\}_{t=1,2,\dots}$ means that
   (a) $\mathrm{E}(X_t)$ is an increasing function of t.
   (b) the variance of X_t is an increasing function of t.
   (c) the time series is stationary.
   (d) there is an increasing correlation among the variables X_t.
2. A convolution of linear filters $\{a_j\} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ and $\{b_k\} = \left(\frac{1}{2}, \frac{1}{2}\right)$ is a filter $\{c_i\}$ equal to:
   (a) $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$.
   (b) $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{6}\right)$.
   (c) $\left(\frac{1}{6}, \frac{2}{3}, \frac{1}{6}\right)$.
   (d) $\left(\frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}\right)$.
3. A weakly stationary process is such that
   (a) all its n-tuples of variables X_t are equal in distribution.
   (b) it has a constant mean and all its autocorrelations are constant.
   (c) it has a constant mean and a constant variance and the covariance $\mathrm{cov}(X_t, X_{t+\tau})$ does not depend on time for any $\tau = 1, 2\dots$.
   (d) all variables X_t are identically distributed.
4. The Autocovariance Function $\mathrm{cov}(X_t, X_{t+\tau})$ of a stationary time series $\{X_t\}_{t=1,2,\dots}$ has the following property:
   (a) It is an even, non-negative, decreasing function of time t.
   (b) It is an even function of lag τ.
   (c) It is a monotonic function of time t.
   (d) It is a bounded function between -1 and 1.
5. The model written as $X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$, where $Z_t \sim WN(0, \sigma^2)$, represents
   (a) AR(2).
   (b) MA(2).
   (c) ARMA(2,2).
   (d) ARMA(1,2).
6. The Autocorrelation Function of the process $X_t = 0.5X_{t-1} + Z_t$, where $Z_t \sim WN(0, \sigma^2)$, at lag $\tau = 2$ is equal to
   (a) $\rho(2) = 0.4$.
   (b) $\rho(2) = 0.5$.
   (c) $\rho(2) = 0.25$.
   (d) $\rho(2) = -0.25$.

7. The ACF of MA(q) of the form $\frac{1}{q+1}\sum_{k=0}^{q} Z_{t-k}$, where Z_t is a White Noise random variable, is

   (a) $\rho(\tau) = \begin{cases} \frac{q-\tau}{q+1} & \text{for} \quad \tau = 0, 1, ..., q \\ 0 & \text{for} \quad \tau > q \end{cases}$

   (b) $\rho(\tau) = \begin{cases} \frac{q-1+\tau}{q+1} & \text{for} \quad \tau = 0, 1, ..., q \\ 0 & \text{for} \quad \tau > q \end{cases}$

   (c) $\rho(\tau) = \begin{cases} \frac{q+1-\tau}{q+1} & \text{for} \quad \tau = 0, 1, ..., q \\ 0 & \text{for} \quad \tau > q \end{cases}$

   (d) $\rho(\tau) = \begin{cases} \frac{\tau}{q+1} & \text{for} \quad \tau = 0, 1, ..., q \\ 0 & \text{for} \quad \tau > q \end{cases}$

8. An explosive AR(1) process $X_t = 1.2X_{t-1} + Z_t$, where $Z_t \sim WN(0, \sigma^2)$,

   (a) is causal and invertible.
   (b) is not causal.
   (c) is stationary.
   (d) has constant mean but non-constant variance.

9. The ACF of the process $X_t - 0.5X_{t-1} = Z_t + 0.7Z_{t-1}$ at lag 1 is $\rho(1) = 0.74$. The ACF at lag two and at lag three is, respectively, equal to:

   (a) $\rho(2) = 0.25$, $\rho(3) = 0.125$.
   (b) $\rho(2) = 0.49$, $\rho(3) = 0.343$.
   (c) $\rho(2) = 0.518$, $\rho(3) = 0.3626$.
   (d) $\rho(2) = 0.37$, $\rho(3) = 0.185$.

10. The Ljung-Box Q statistic used in the Minitab ARIMA modelling allows you

   (a) to test the null hypothesis that the autocorrelations of the residuals for all lags up to lag k are equal to zero.
   (b) to test the null hypothesis that the model parameters are non-significant.
   (c) to test the null hypothesis that the autocorrelations of the residuals are between -1 and 1.
   (d) to test the null hypothesis that the variance of the residuals is constant.