**Part 2** We use the notation as in the lecture notes, specifically

$$\phi(z) = 1 - \phi_1 z - ... - \phi_p z^p$$
$$\theta(z) = 1 + \theta_1 z + ... + \theta_q z^q$$
$$Z_t \sim WN(0, \sigma^2)$$

*Choose a correct answer for each of the following problems.*

1. There is a stationary solution $X_t$ to $\phi(B)X_t = \theta(B)Z_t$ process if and only if
   (a) $\phi(z) \neq 0$ for all $|z| = 1$.
   (b) $\phi(z) = 0$ for all $|z| = 1$.
   (c) $\theta(z) \neq 0$ for all $|z| = 1$.
   (d) $\theta(z) = 0$ for all $|z| = 1$.

2. An $ARMA(p, q)$ process is causal if and only if
   (a) $\phi(z) = 0$ only for $|z| > 1$.
   (b) $\phi(z) = 0$ only for $|z| < 1$.
   (c) $\theta(z) = 0$ only for $|z| > 1$.
   (d) $\theta(z) = 0$ only for $|z| < 1$.

3. The process $X_t - 0.7X_{t-1} + 0.1X_{t-2} = Z_t + 0.5Z_{t-1}$ is
   (a) causal but not invertible.
   (b) neither causal nor invertible.
   (c) causal and invertible.
   (d) not causal but invertible.

4. The following is true for ACF and PACF of AR(p):
   (a) ACF tails off and PACF cuts of after lag q.
   (b) ACF cuts off after lag p and PACF tails off.
   (c) ACF cuts off after lag p and PACF cuts of after lag q.
   (d) ACF tails off and PACF cuts off after lag p.

5. PACF of the process $X_t - 0.7X_{t-1} + 0.1X_{t-2} = Z_t$ is equal to:
   (a) $\phi_{\tau\tau} = 0.7^\tau$ for $\tau = 0, 1, 2, ....$
   (b) $\phi_{11} = \frac{7}{11}$, $\phi_{22} = -0.1$, $\phi_{\tau\tau} = 0$ for $\tau > 2$.
   (c) $\phi_{11} = \frac{7}{11}$, $\phi_{\tau\tau} = 0$ for $\tau > 1$.
   (d) $\phi_{11} = 0.7$, $\phi_{22} = -0.1$, $\phi_{\tau\tau} = 0$ for $\tau > 2$.

6. The process $X_t - 0.5X_{t-1} = Z_t - 1.3Z_{t-1} + 0.4Z_{t-2}$ represents
   (a) $ARMA(1, 2)$.
   (b) $AR(1)$.
   (c) $MA(1)$.
   (d) $ARMA(1, 1)$.

7. The following is true for ACF and PACF of $ARMA(1,1)_4$:
   (a) ACF cuts off after lag 4 and PACF tails off.
   (b) ACF cuts off after lag 4 and PACF tails off at lags $4k$, $k = 1, 2, ....$
   (c) Both ACF and PACF tail off at lags $4k$, $k = 1, 2, ....$
   (d) Both ACF and PACF cut off at lag 4.

   and the values of ACF and PACF are zero at non-seasonal lags, i.e., at $\tau \neq 4k$.

8. ACF of the process $X_t = Z_t + 0.5Z_{t-4}$ is
   (a) $\rho(0) = 1,\ \rho(\pm 4) = 0.5^4,\ \rho(\tau) = 0$ otherwise.
   (b) $\rho(0) = 1,\ \rho(\pm 4) = 0.5,\ \rho(\tau) = 0$ otherwise.
   (c) $\rho(0) = 1,\ \rho(\pm 4) = 0.25,\ \rho(\tau) = 0$ otherwise.
   (d) $\rho(0) = 1,\ \rho(\pm 4) = 0.4,\ \rho(\tau) = 0$ otherwise.

9. A time series $X_t$ has the following ACF

$$\begin{aligned}
&\rho(12j) = 0.8^j \\
&\rho(12j \pm 1) = 0.4 \times 0.8^j \text{ for } j = 0, \pm 1, \pm 2, ... \\
&\rho(\tau) = 0 \text{ otherwise}
\end{aligned}$$

   Which of the following models it could be the ACF of?
   (a) $X_t + 0.8X_{t-12} = Z_t - 0.5Z_{t-1}$.
   (b) $X_t - 0.5X_{t-12} = Z_t + 0.8Z_{t-1}$.
   (c) $X_t - 0.8X_{t-12} = Z_t + 0.4Z_{t-1}$.
   (d) $X_t - 0.8X_{t-12} = Z_t + 0.5Z_{t-1}$.

10. The process $(1 - B^7)(1 - B)X_t = (1 + 0.9B^7)(1 + 0.5B)Z_t$ is
    (a) $ARIMA(0, 0, 1) \times (1, 1, 1)_7$.
    (b) $ARIMA(0, 1, 0) \times (1, 1, 1)_7$.
    (c) $ARIMA(1, 1, 1) \times (1, 1, 1)_7$.
    (d) $ARIMA(0, 1, 1) \times (0, 1, 1)_7$.