Assignment #5

In problems 1) – 3) $H = -\Delta + V$ on $\mathbb{Z}^d$, where $\{V(x)\}$ are iid random variables.

① $\sigma(H) = \sigma_{ess}(H) = \{ a + b \mid a \in \sigma(-\Delta) = [0, 4d], \ b \in \text{supp } V(0) \}$

② if $V(0)$ has bounded density, then

$$\mathbb{E} \, | (H-z)^{-1} (x,y) |^s \leq C_s, \qquad 0 < s < 1,$$

and moreover the same estimate holds when the expectation is taken only over $V(x), V(y)$ (conditionally on the rest).

③ Suppose $\mathbb{P}\{ |V(x) - a| \leq \epsilon \} \leq C_0 \epsilon^\alpha.$ Then

$$\{ \lambda \in \mathbb{R} \mid \exists C, c, s > 0: \sup_{0 < \eta < 1} \mathbb{E} \, | G_{\lambda + i\eta} [H] (0,x) |^s \leq C e^{-c|x|} \}$$

is open.

④ $$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} E & F \\ G & H \end{pmatrix} \qquad \text{where}$$

$$E = (A - BD^{-1}C)^{-1}, \qquad G = -D^{-1}C(A-BD^{-1}C)^{-1}$$