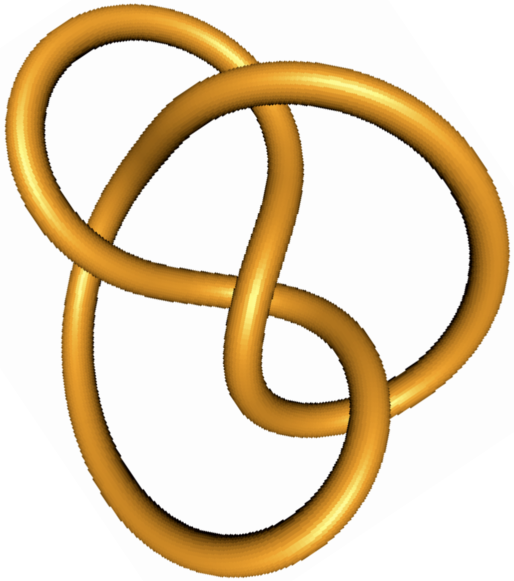
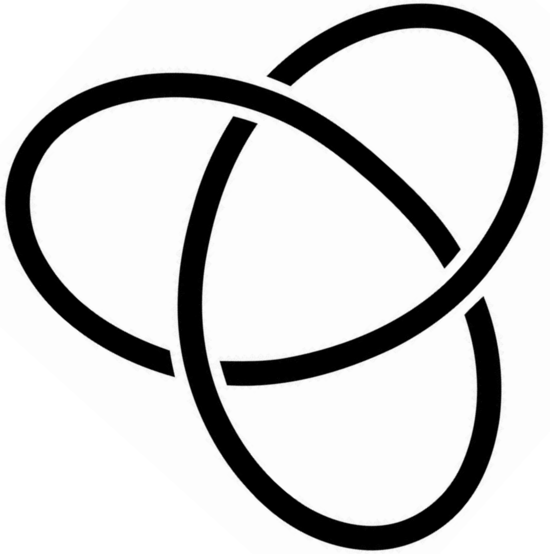


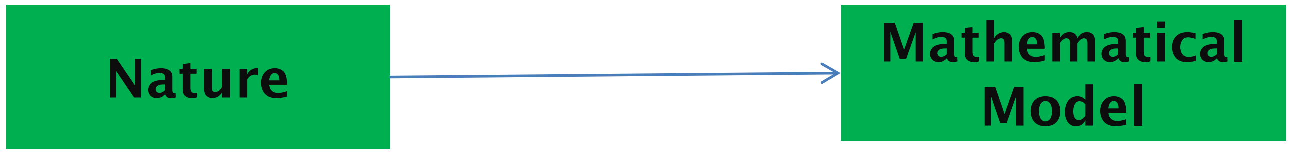


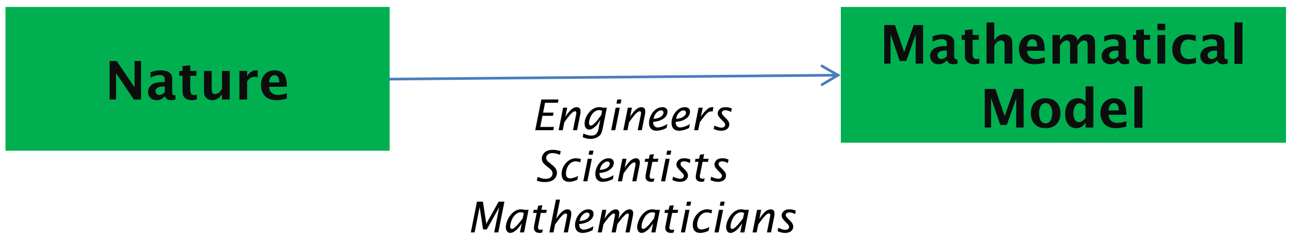
Knots

(and how to tame them…)

By Aryan Ghobadi

Royal Institute Masterclass, 1st Feb 2020


Nature
Mathematical Model


Nature
Engineers
Scientists
Mathematicians
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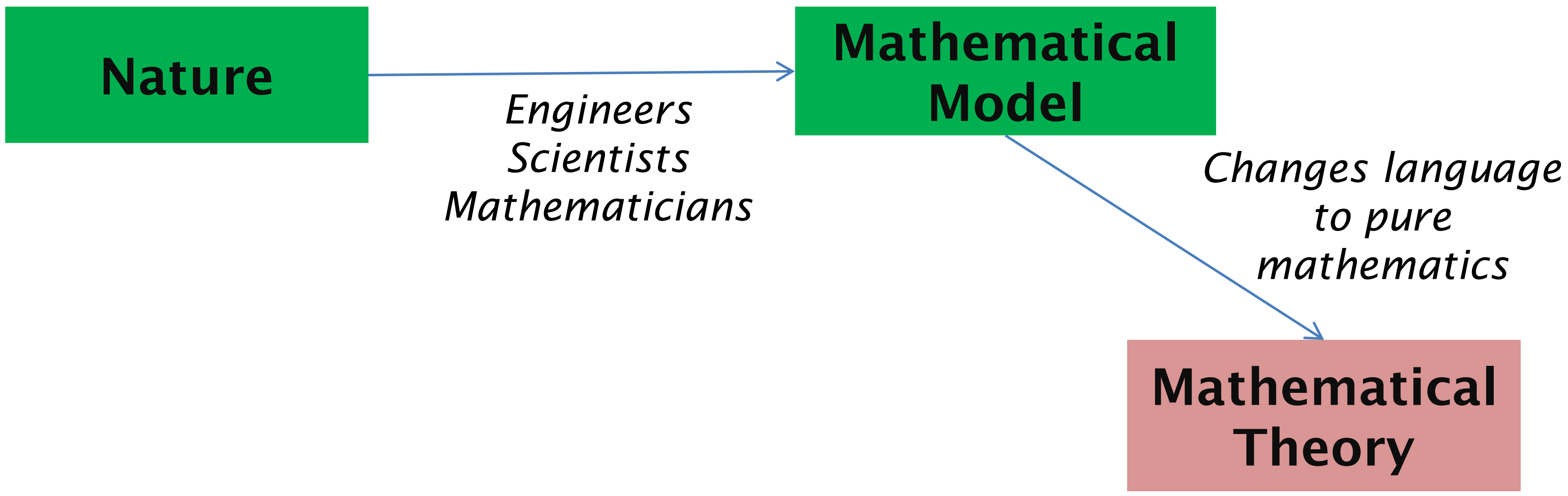
Nature
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Changes language
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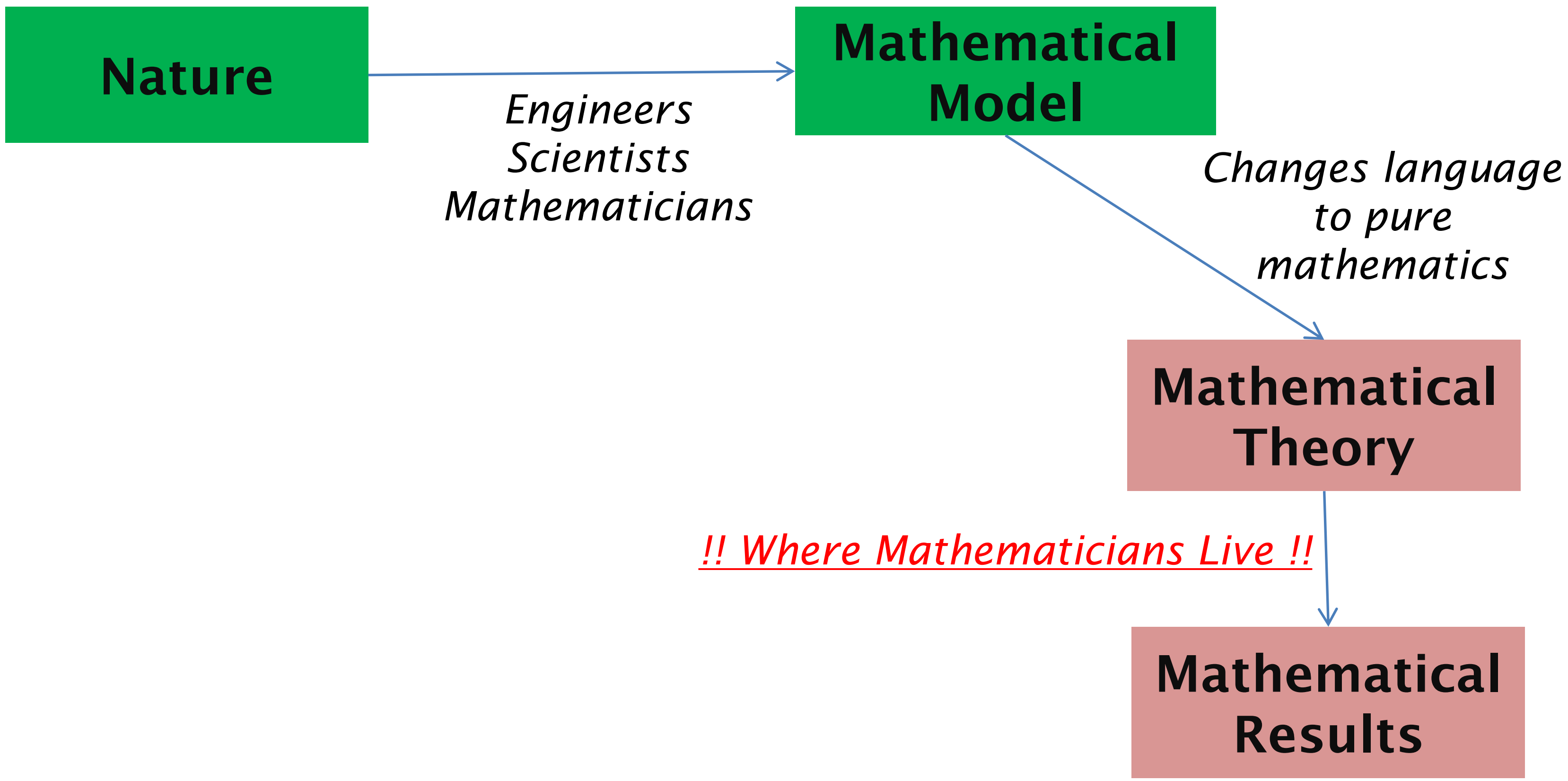
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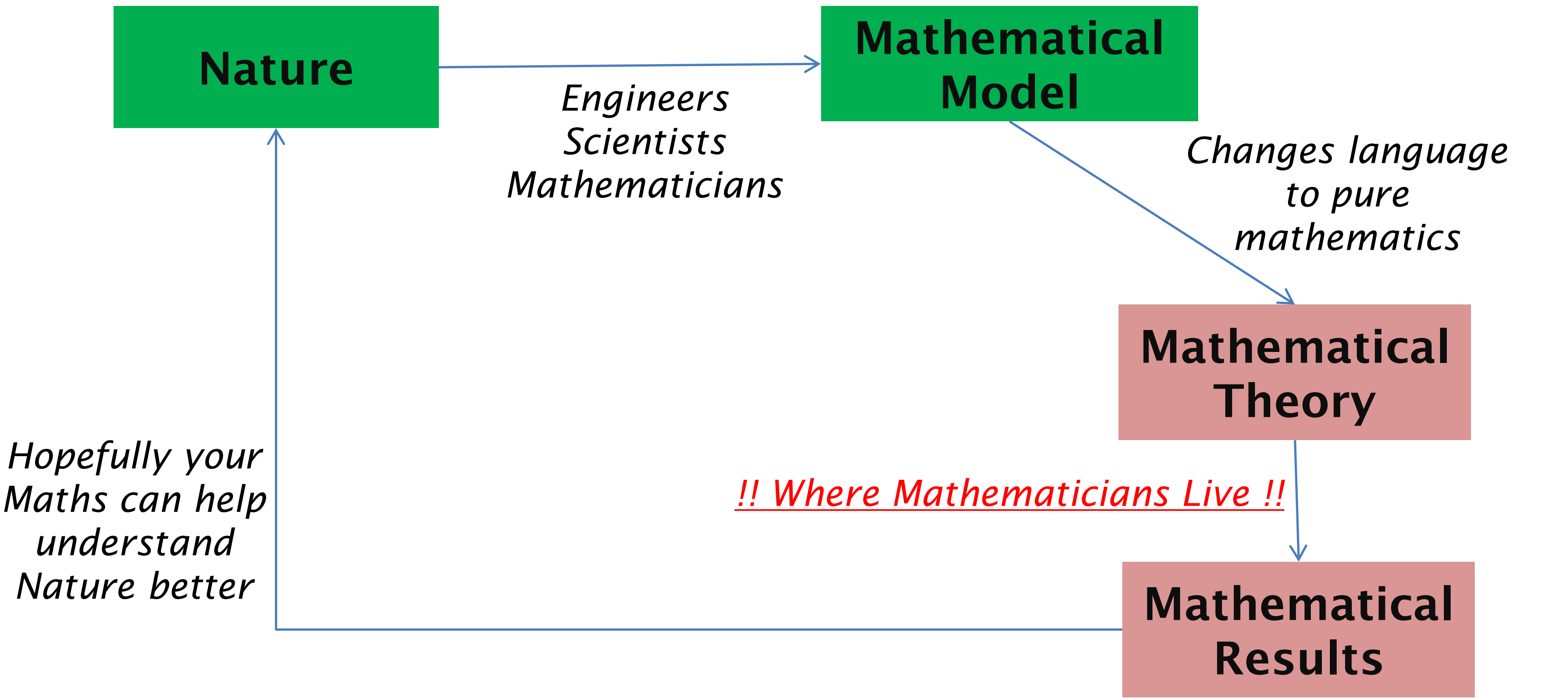
!! Where Mathematicians Live !!
Mathematical Results



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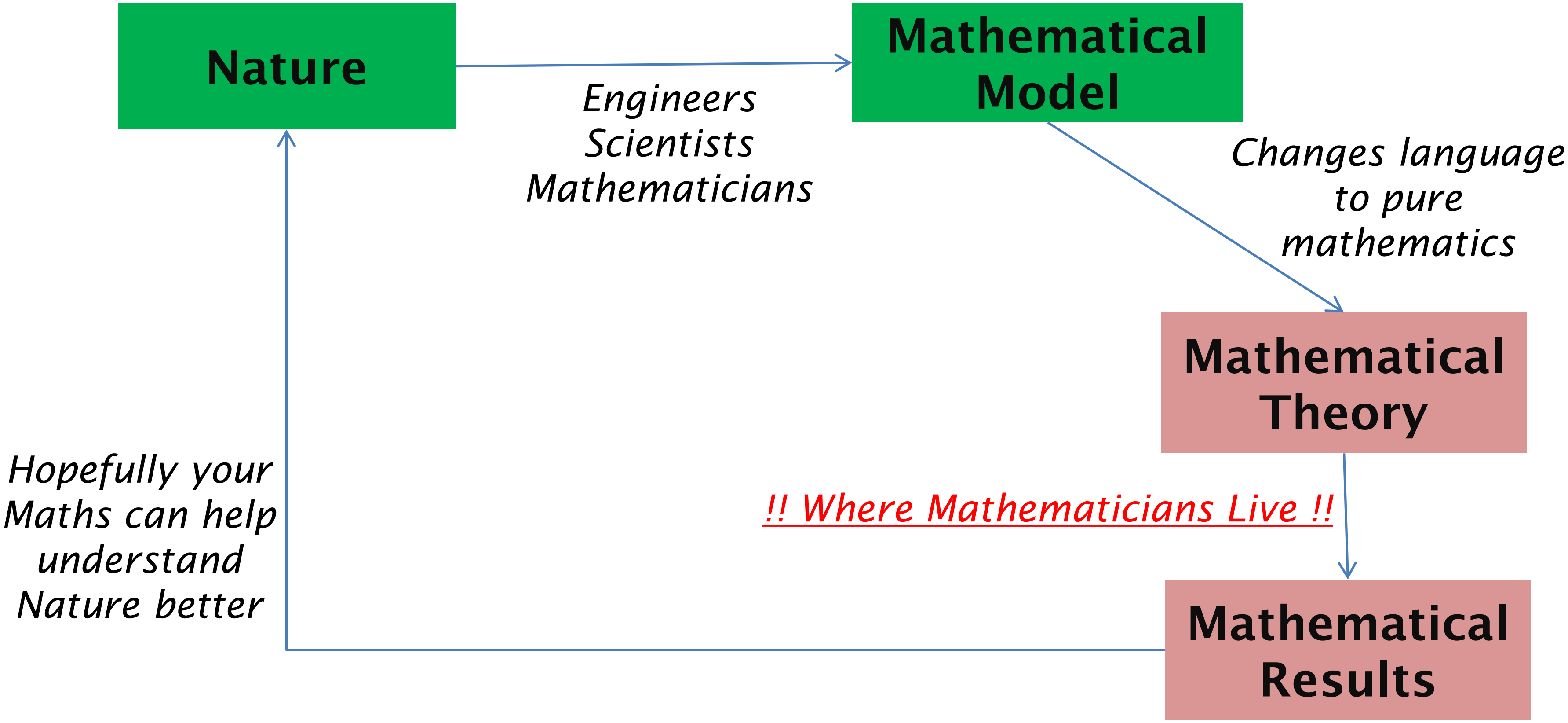
Mathematical Results
Hopefully your Maths can help understand Nature better



- *Mathematical results might not solve the big problem at first but add to overall Mathematical knowledge*
- *Good mathematics should connect with other good mathematics!*

What are Knots?

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- A closed “line” in 3 dimensional space, without intersection

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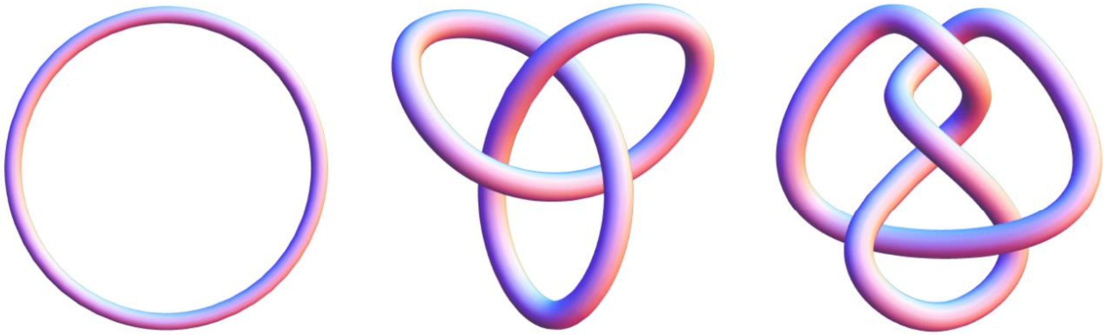

(a) Unknot (b) Trefoil (c) Figure eight

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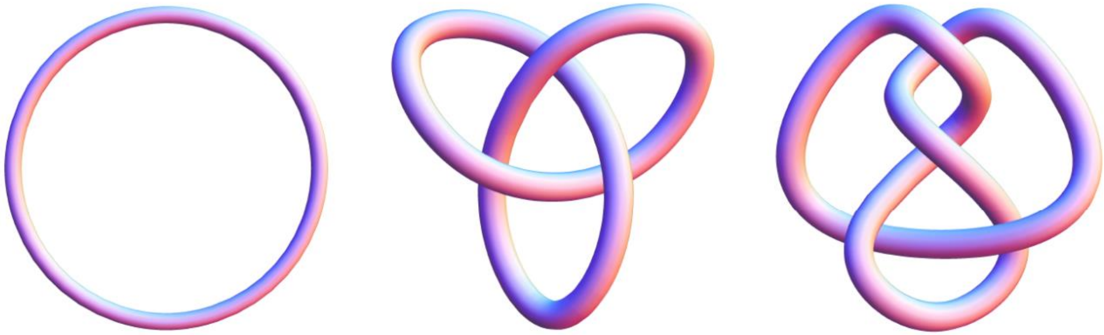

(a) Unknot (b) Trefoil (c) Figure eight

- **More formally:**

A “smooth” function $f:[0,1] \to \mathbb{R}^3$ such that $f(0) = f(1)$ and that’s the only case where $f(x) = f(y)$, for $x \neq y$.

How do we look at Knots?

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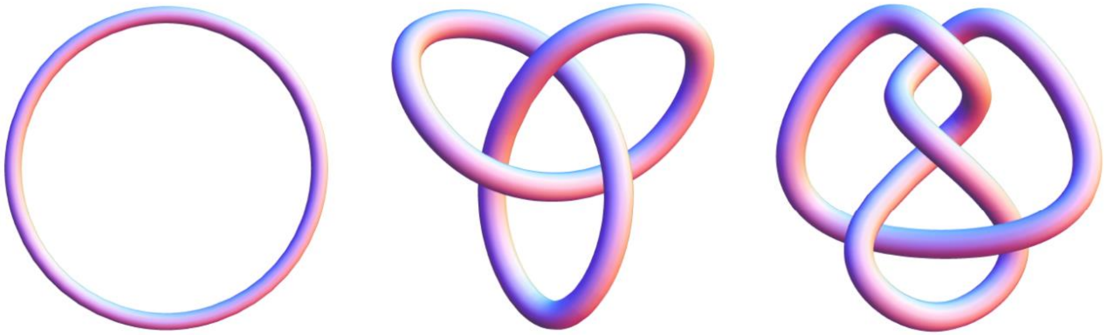
- As 2 dimensional diagrams



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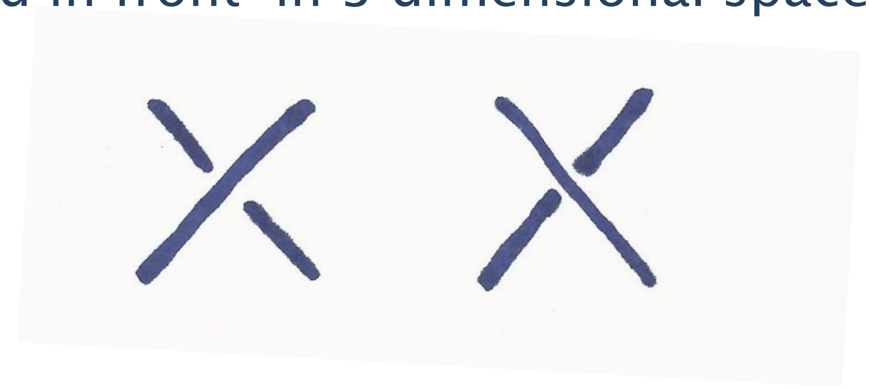
- Crossing behind and in front in 3 dimensional space are represented as



Topological View

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- We care about the “Topology” of Knots.
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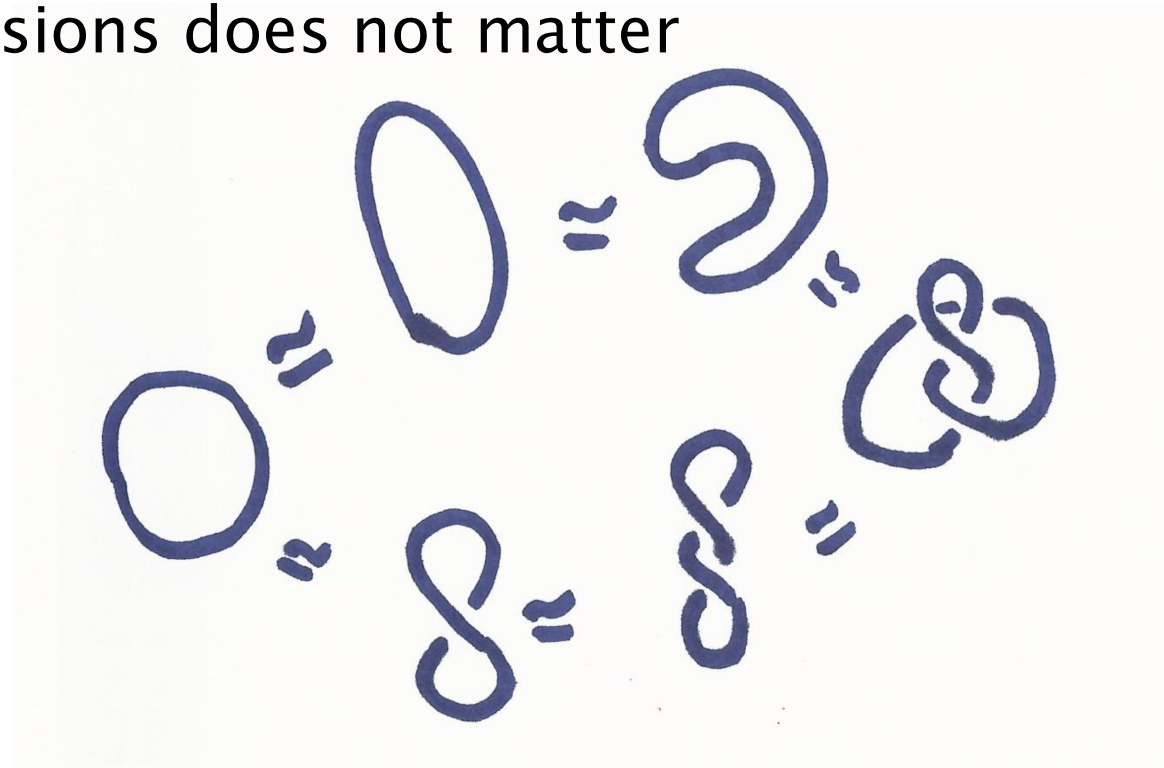

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- **More formally: (Ambient Isotopy)**

Given two knots $k, \bar{k}: [0,1] \to \mathbb{R}^3$, there exists a continuous map

$$F: \mathbb{R}^3 \times [0,1] \to \mathbb{R}^3$$

Such that $F(k(x), 0) = k(x)$ and $F(k(x), 1) = \bar{k}(x)$.

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- Mathematical idea: Find least <u>Crucial Moves</u>

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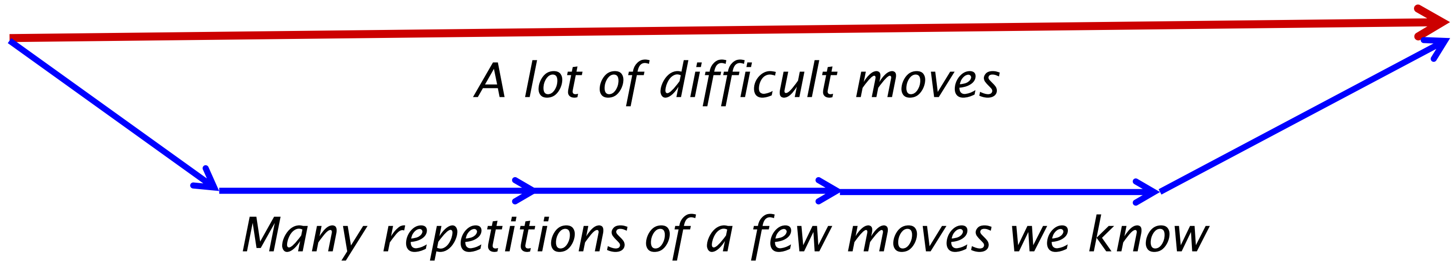

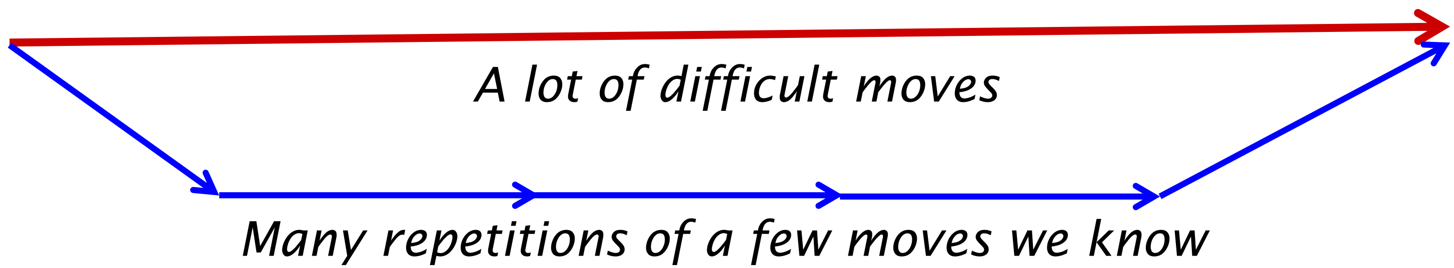
A lot of difficult moves
Many repetitions of a few moves we know

Reidmeister Moves

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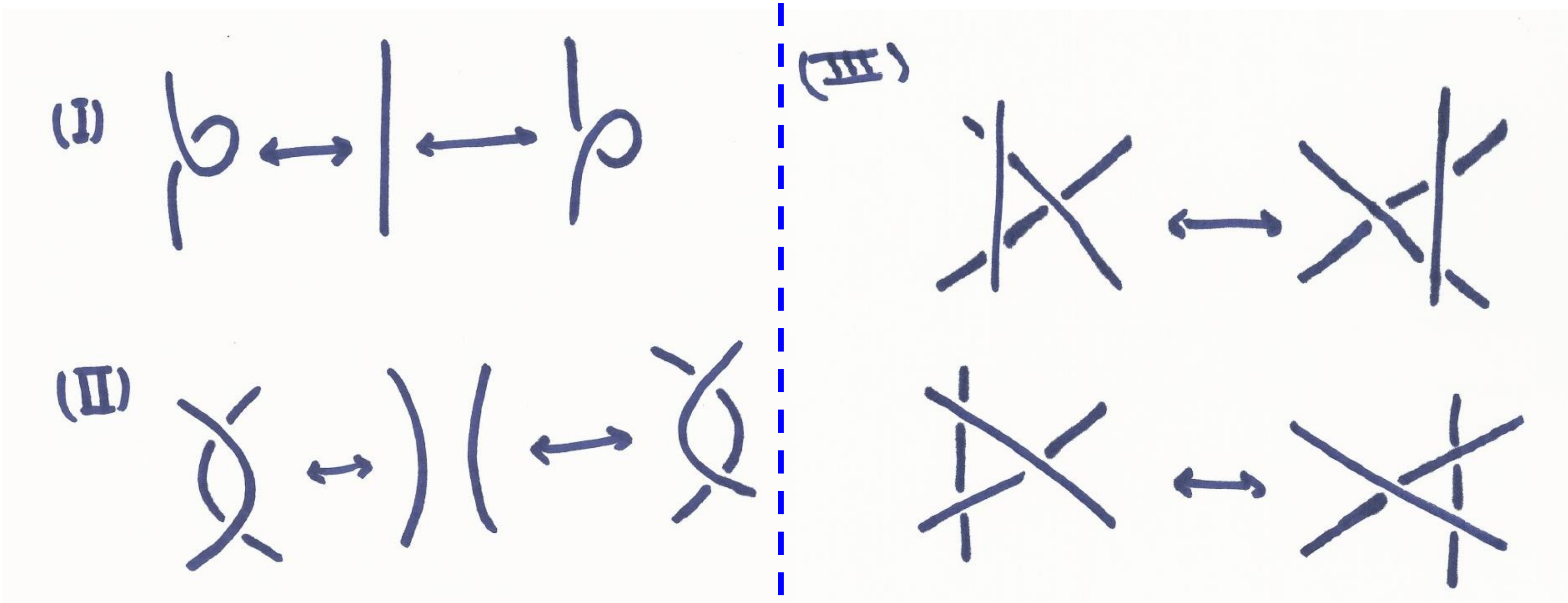

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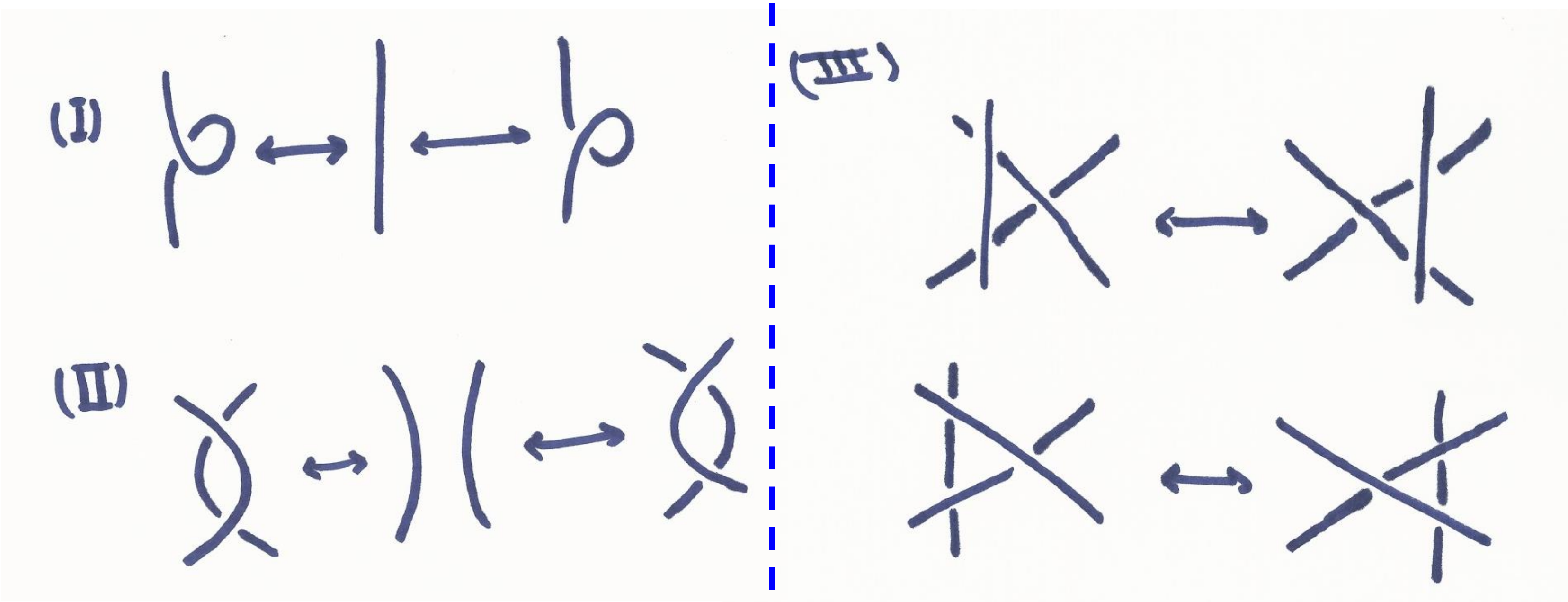
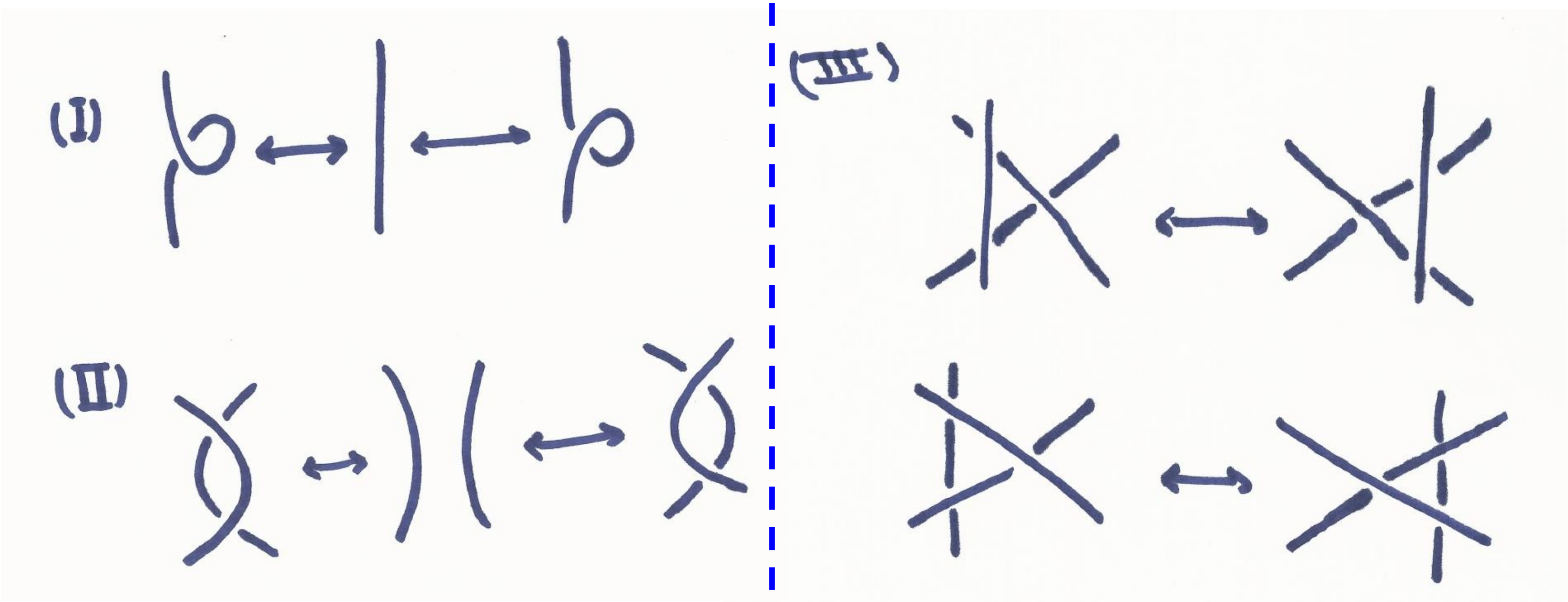


- **More formally: (Reidmeister's Theorem, 1927)**

Given two knots $k, \bar{k}: [0,1] \rightarrow \mathbb{R}^3$, they are equivalent if and only if one can be transformed to the other by finitely many Reidmeister moves.

Reidmeister Moves



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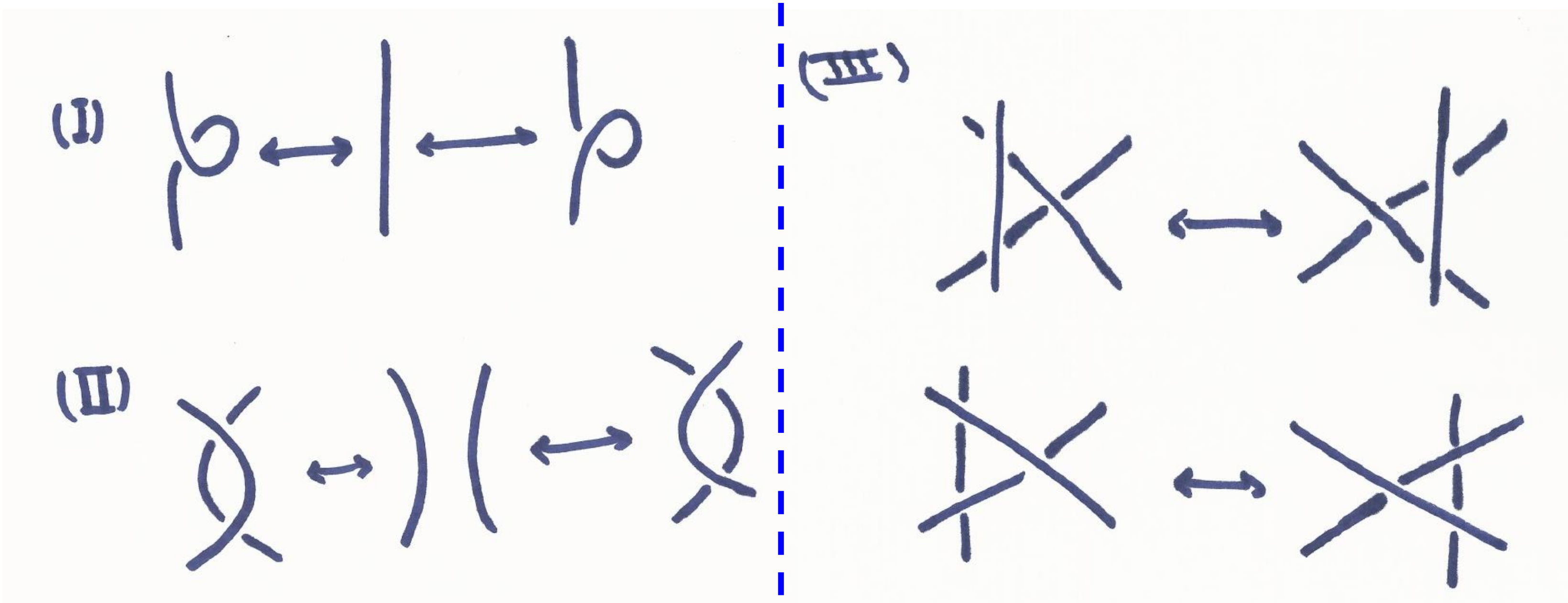

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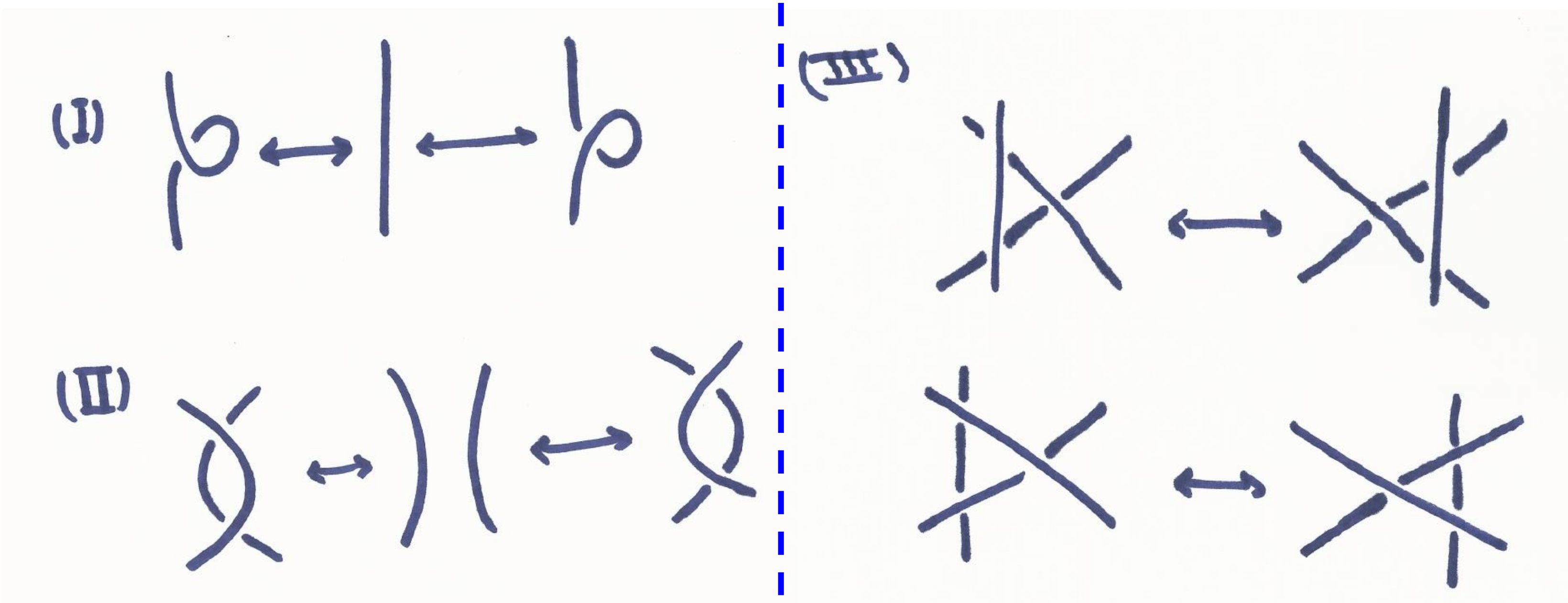

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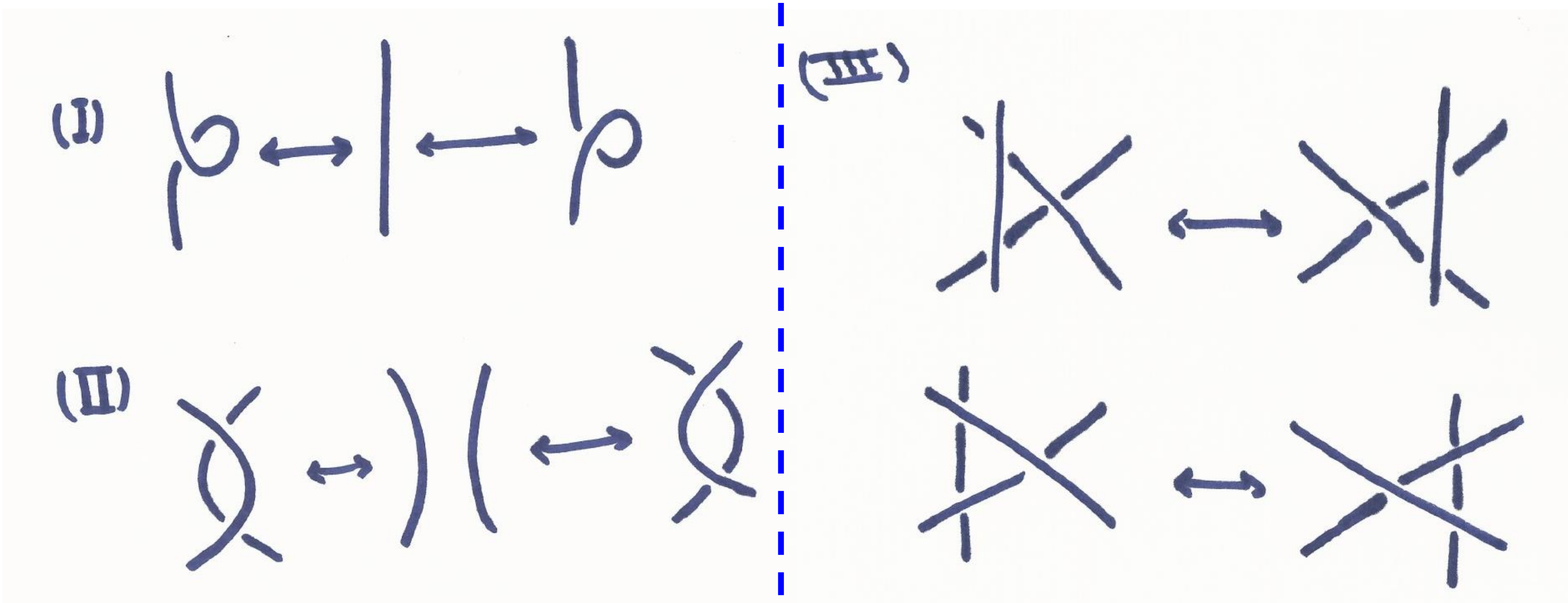

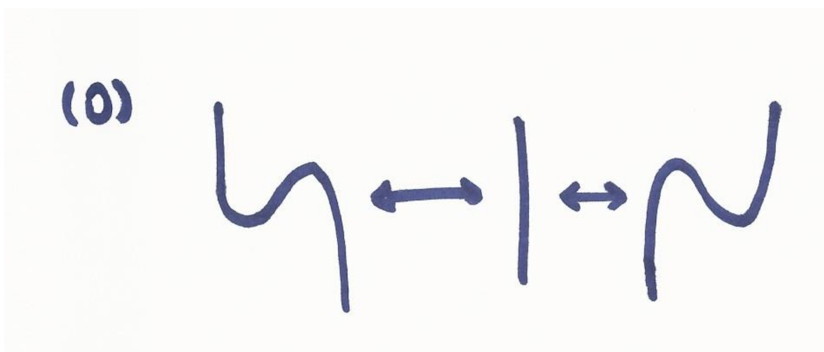

How to tell Knots apart?

Diagrams for the same knot → Reidmesiter → *Same Knot*

How do we tell if knot diagrams are definitely different

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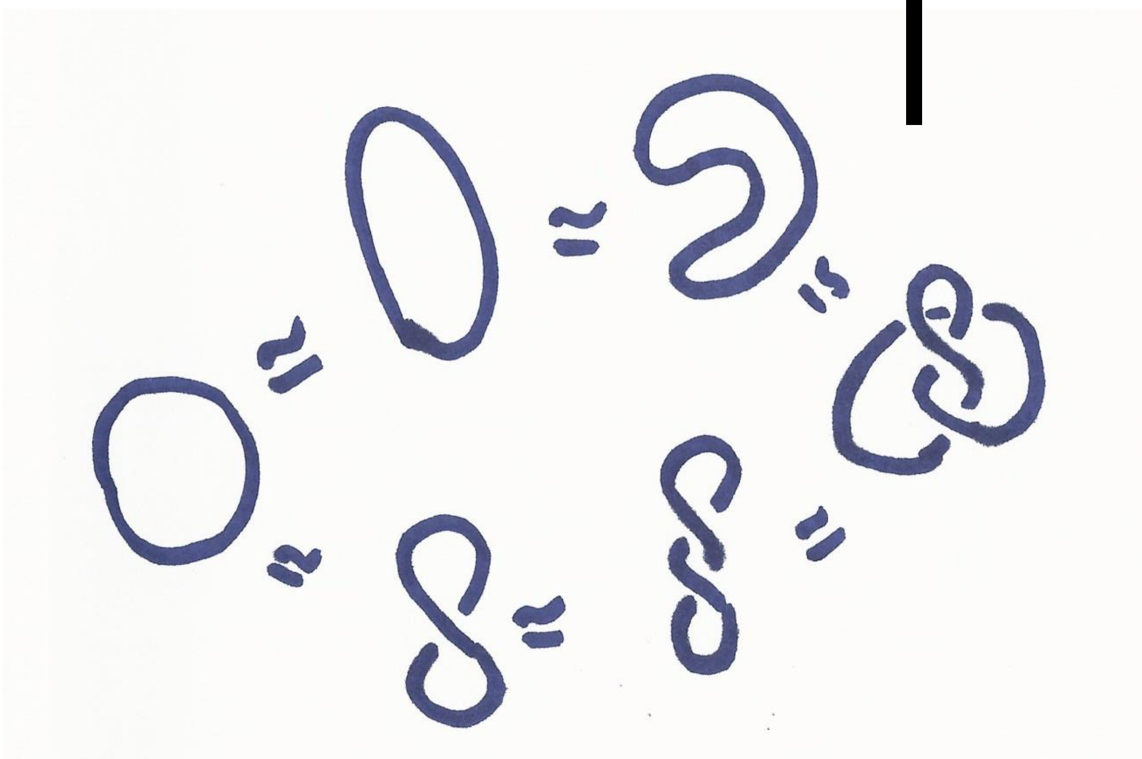

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Answer: (Invariance)

Assign a “number” to each Knot called it’s <u>invariant</u> so that

- Equivalent knots get the same number

Then, if two knots get different numbers, then they’re not equivalent.

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Then, if two knots get different numbers, then they're not equivalent.

- No guarantee that an assignment tells apart all Knots
- The more it does so, the better (a more *coarse* invariant)

Ideas for Invariants

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- Number of Crossings (not an invariant)
- Least Number of crossings possible (invariant)

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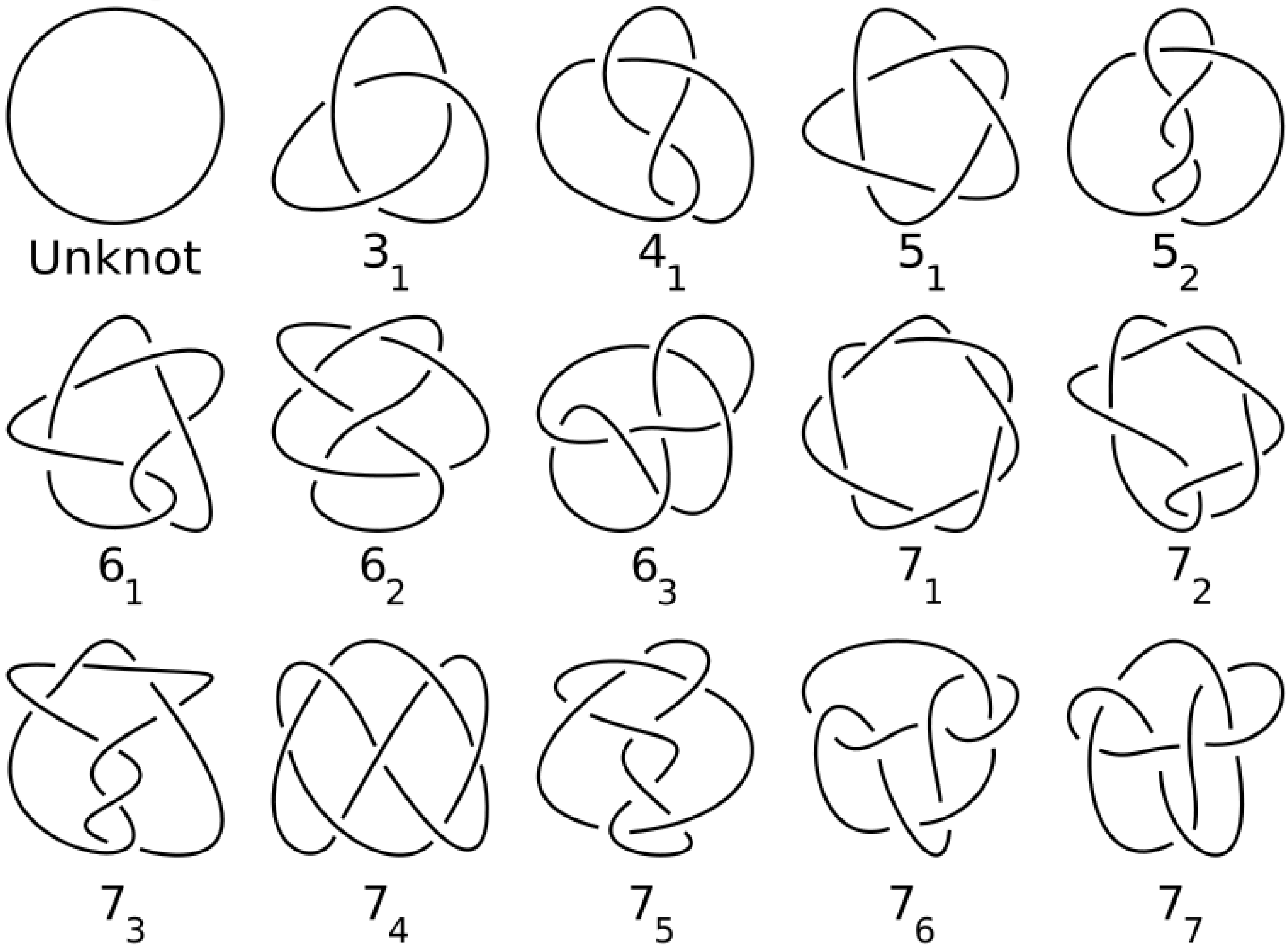

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- Why is any knot with 1 trivial?
- Are there any knots with 2 crossings?

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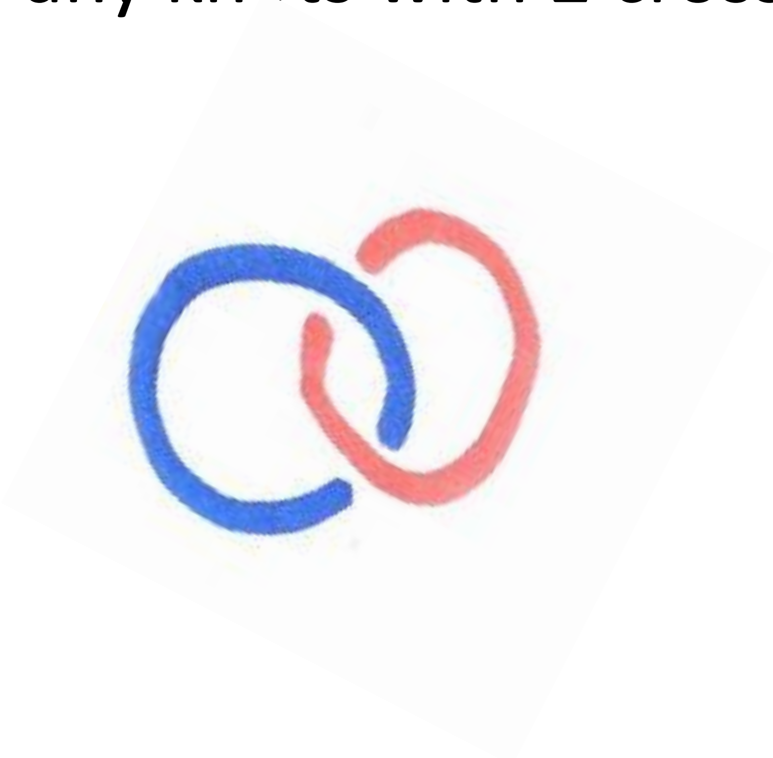

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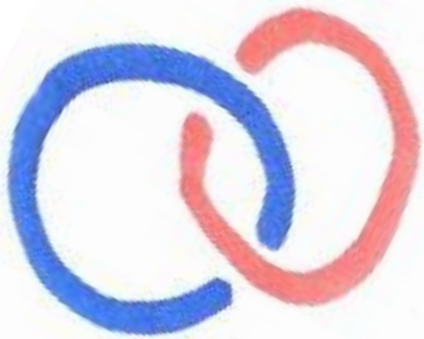

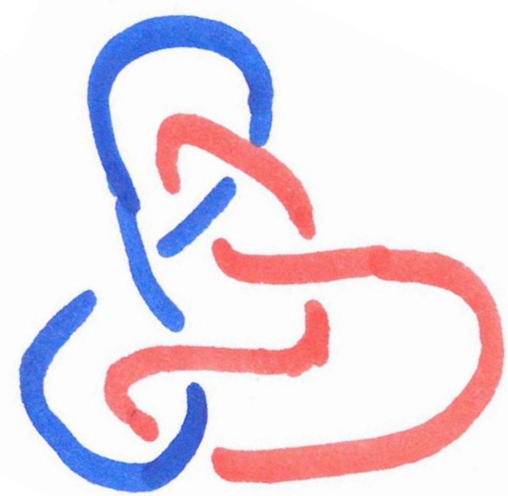

- For a better invariant we need to work with more complicated numbers than $\mathbb{N} = \{1,2,3,4,\ldots\}$

- Assign Polynomials to Knots $\mathbb{Z}[t]$

$$p(t) \in \mathbb{Z}[t] \quad \Rightarrow \quad p(t) = a_n t^n + a_{n-1} t^{n-1} + \cdots + a_1 t + a_0$$

For example $5t^3 + 8t^2 - 2$, $3t^7 + 4t^3$

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Alexander Polynomial

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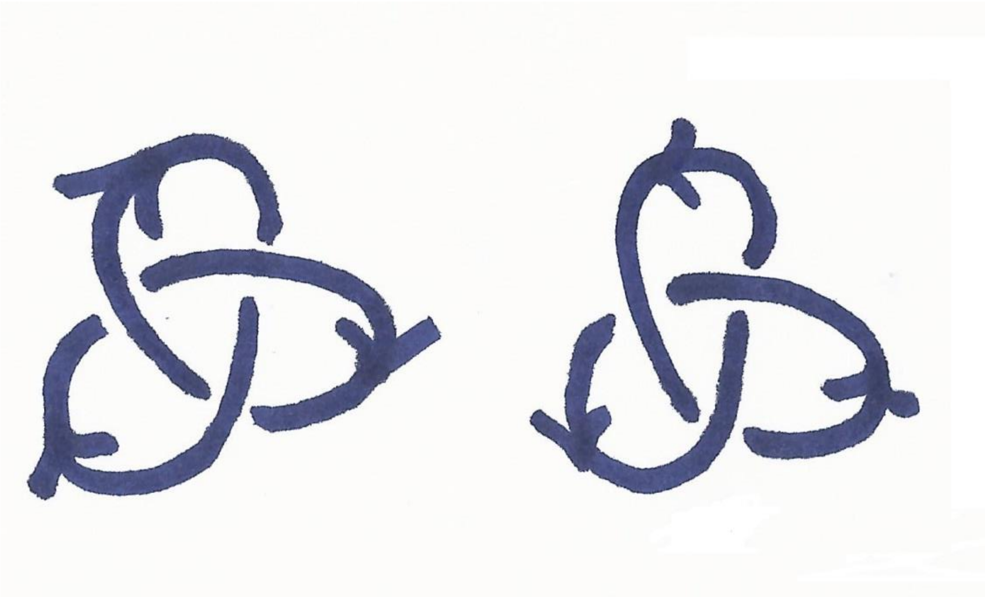

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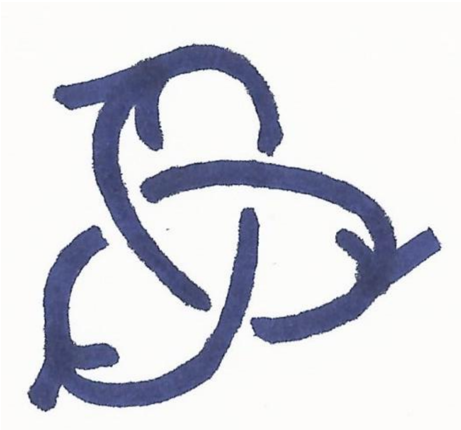

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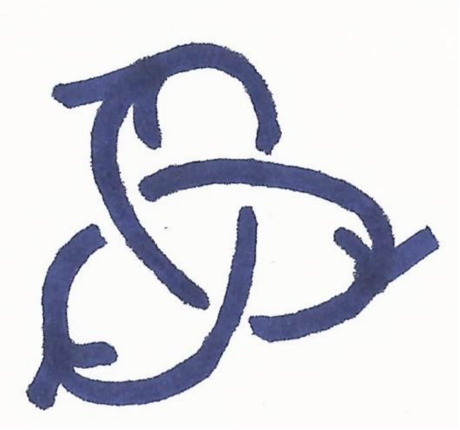
- *(**Euler's Theorem**) A knot diagram with n crossings, divides plane into n+2 regions*

- Name your regions ($r_1, r_2, \cdots, r_{n+2}$)



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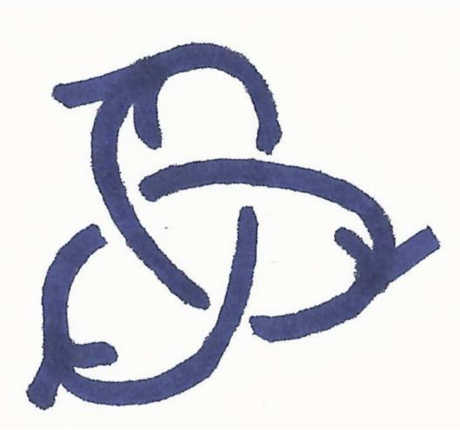

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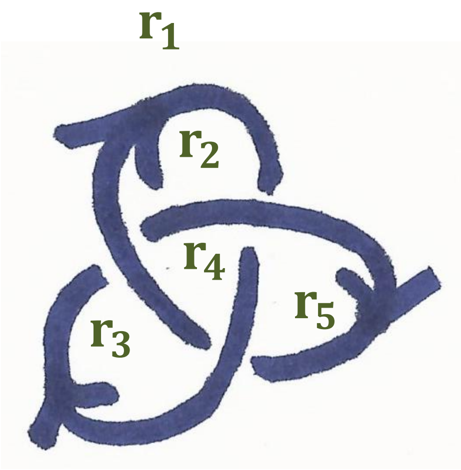

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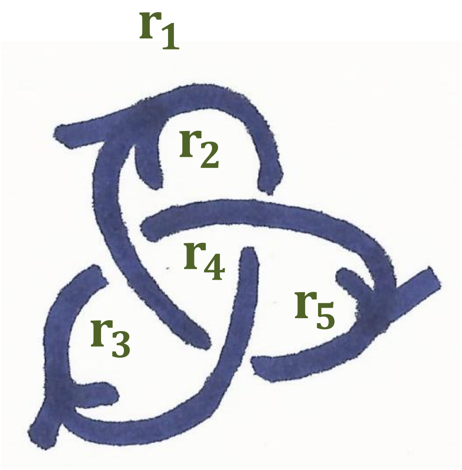

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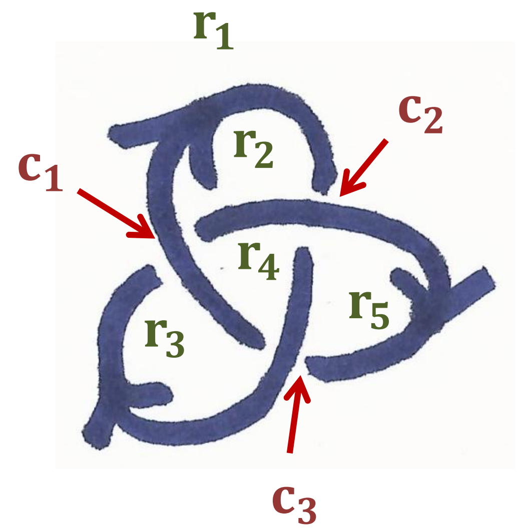

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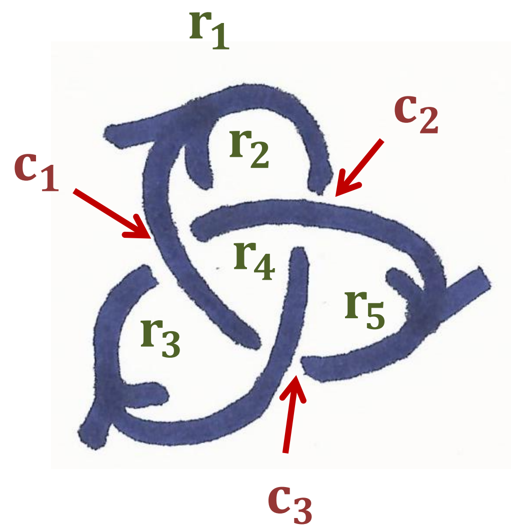

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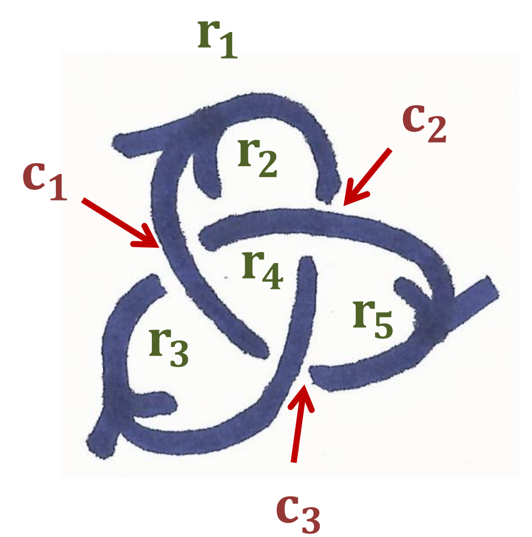
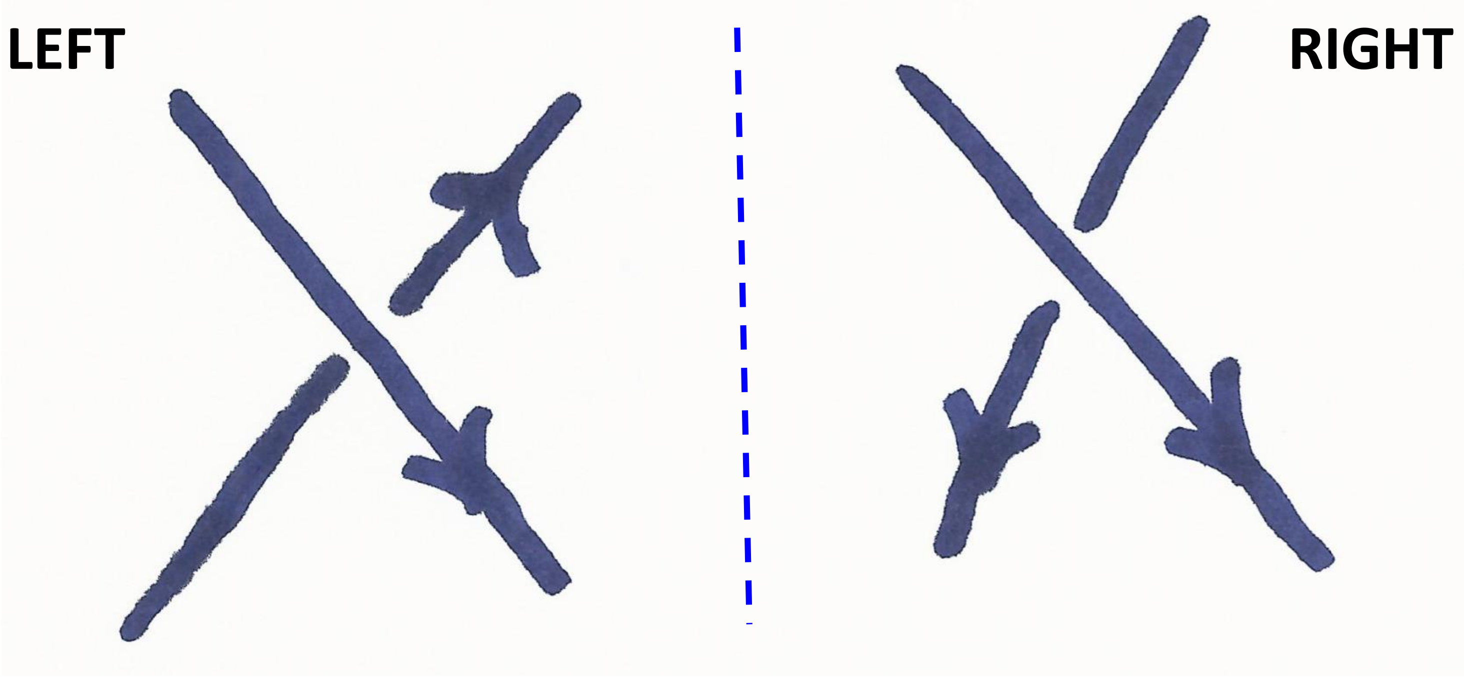
$$M = \begin{matrix} & r_1 & r_2 & r_3 & r_4 & r_5 \\ c_1 & & & & & \\ c_2 & & & & & \\ c_3 & & & & & \end{matrix}$$



Alexander Polynomial

c_i

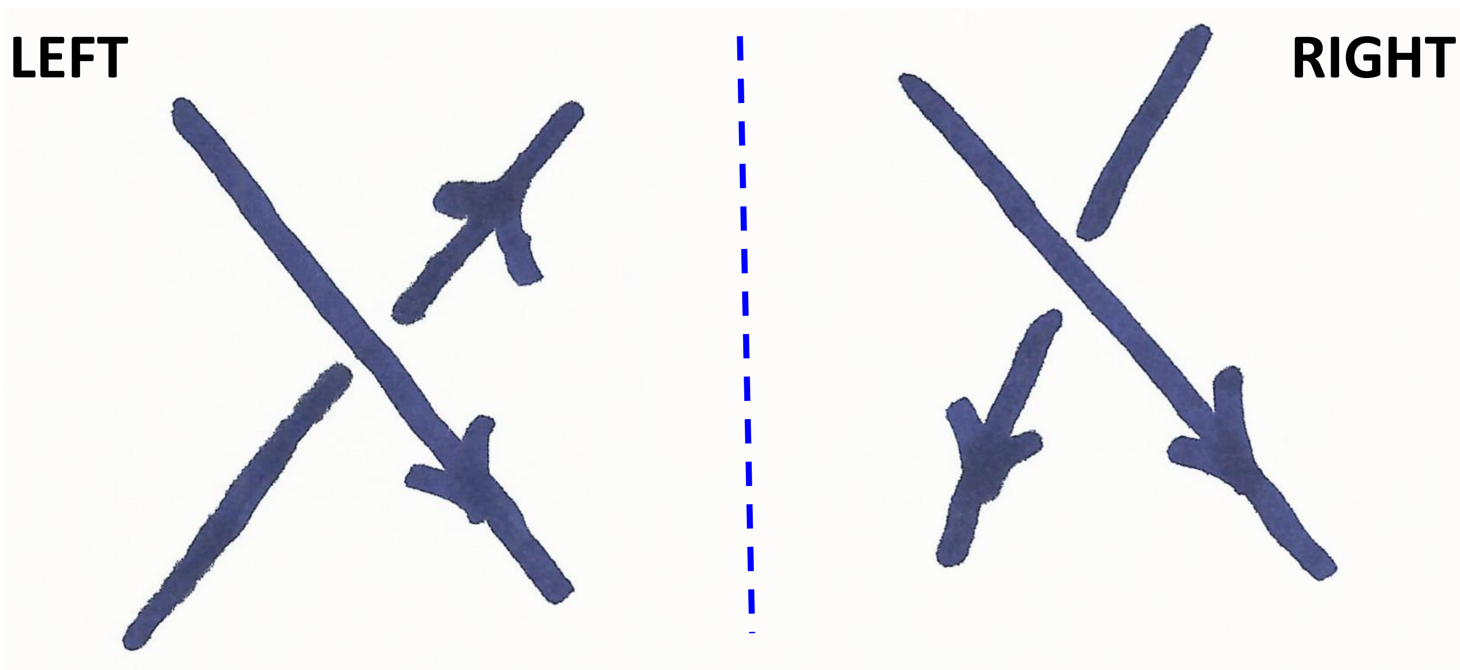


Alexander Polynomial

- Fill Matrix: For each crossing c_i, *(row i)*

\- *use the following pictures to fill the columns of its 4 neighboring region and 0 in all other entries*

c_i



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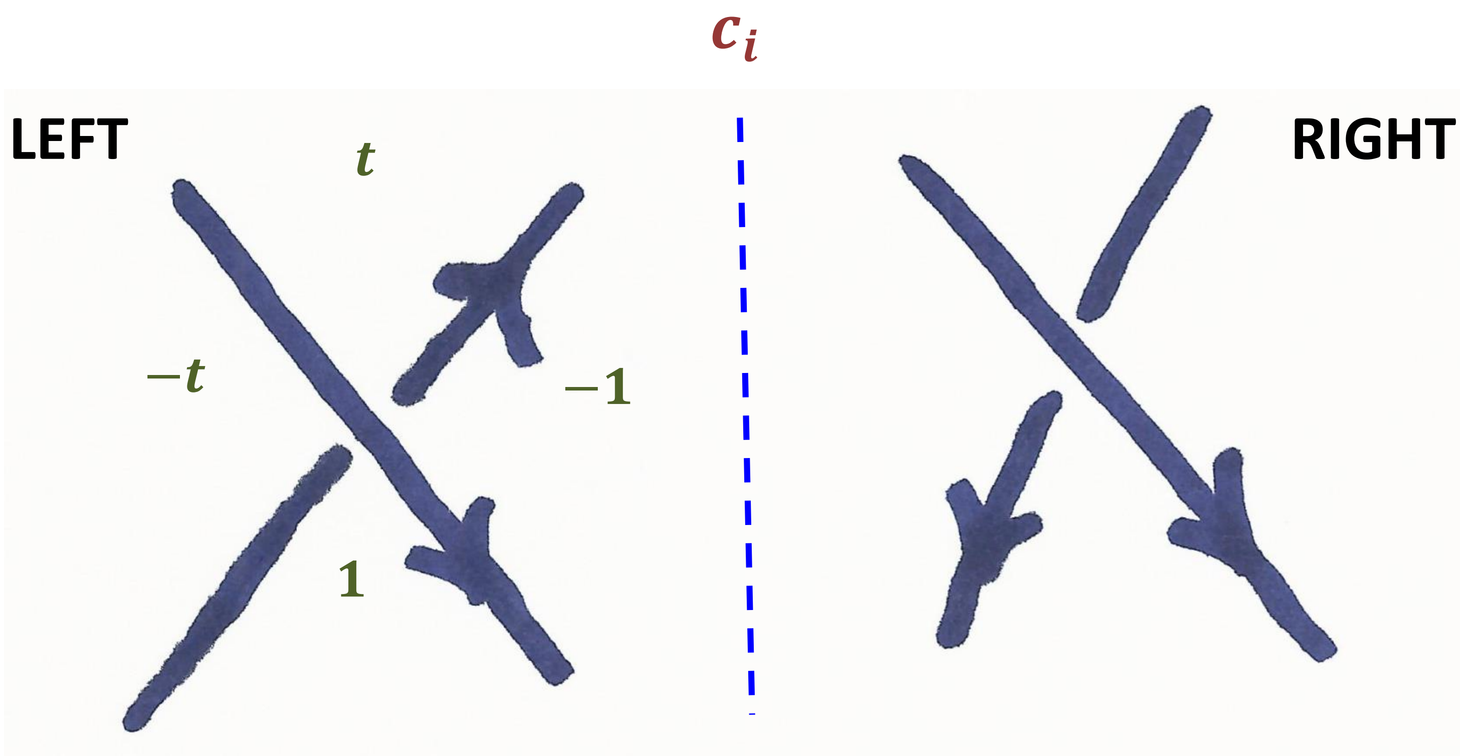

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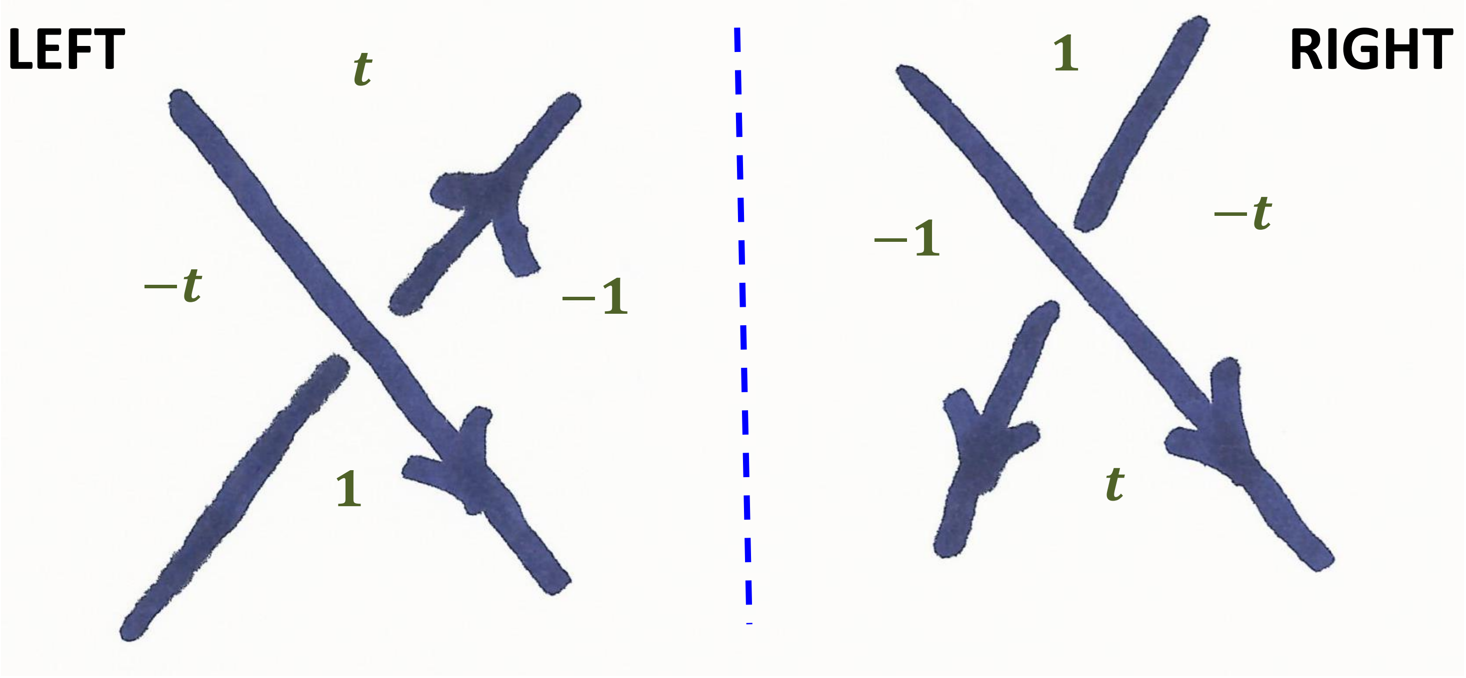

Alexander Polynomial

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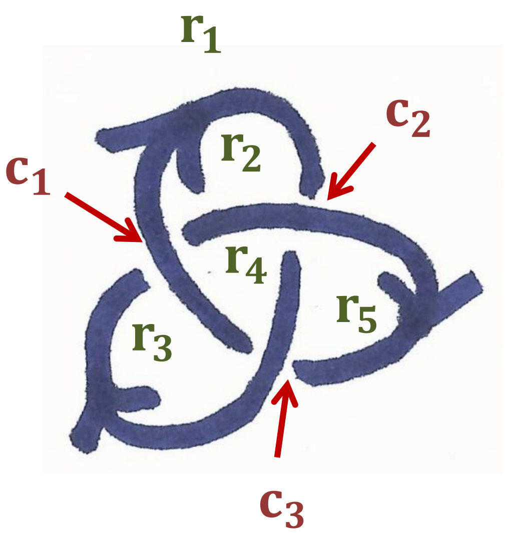

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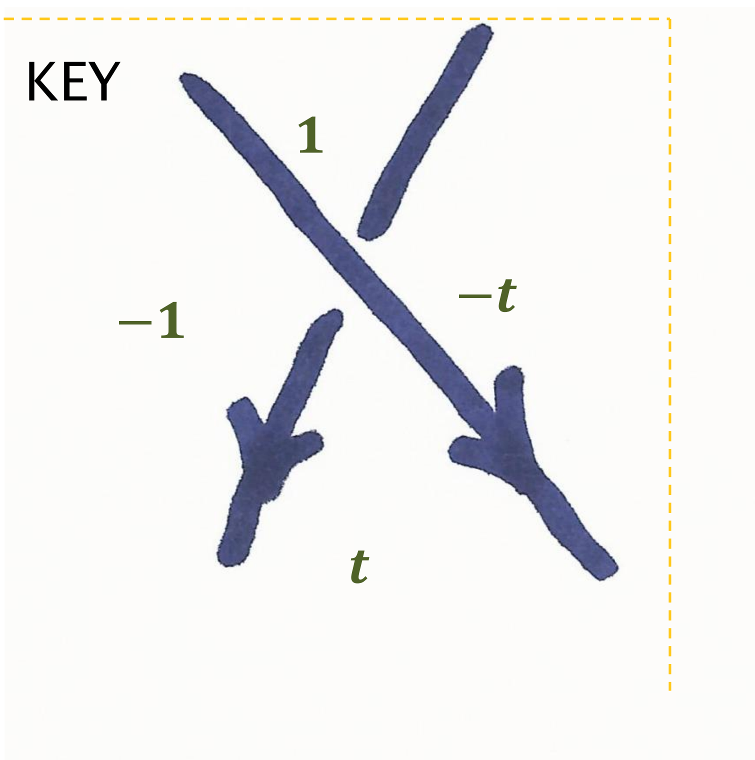

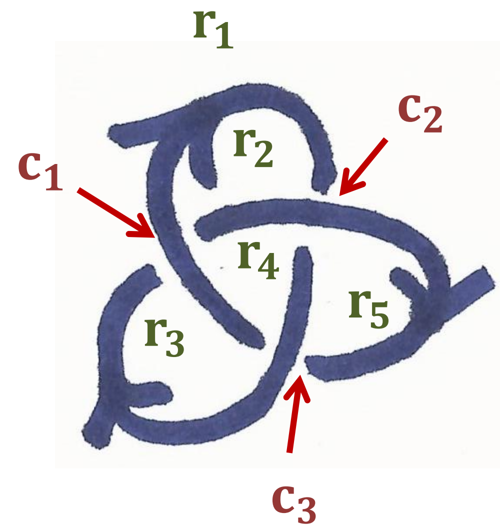

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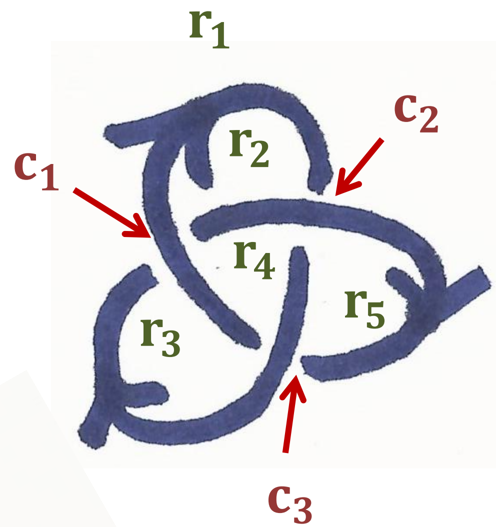
$$M = \begin{matrix} & r_1 & r_2 & r_3 & r_4 & r_5 \\ c_1 & & & & & \\ c_2 & & & & & \\ c_3 & & & & & \end{matrix}$$



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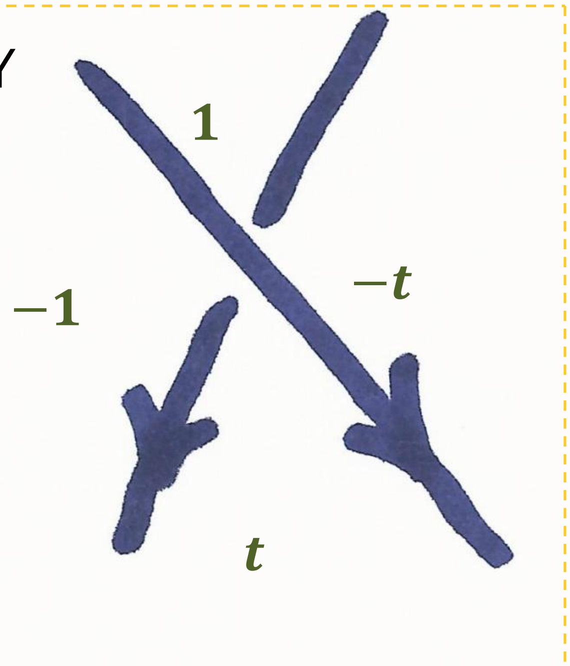
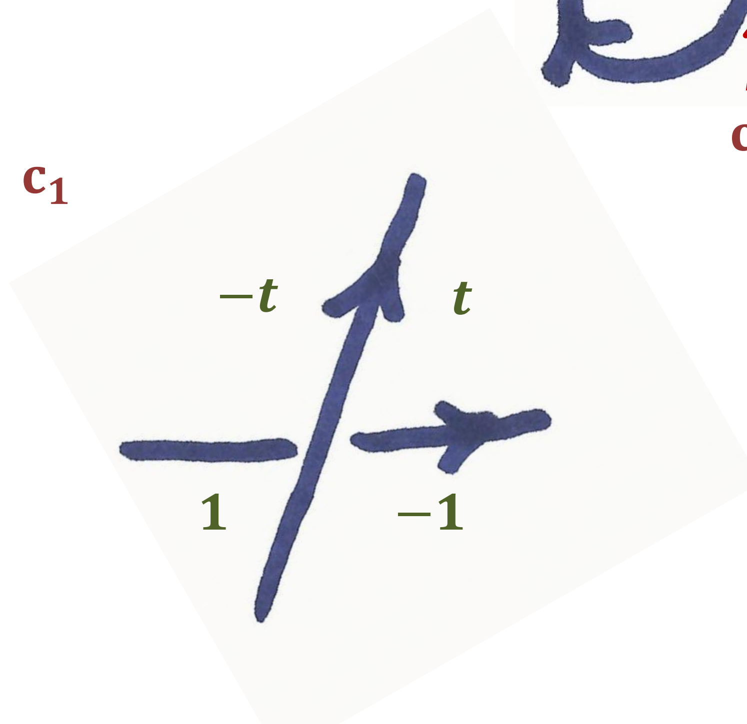


c1



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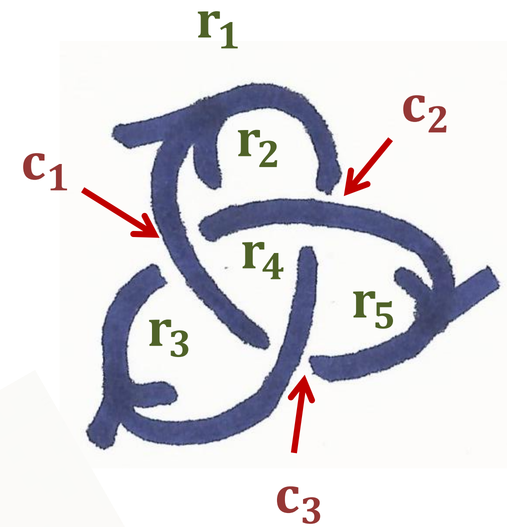

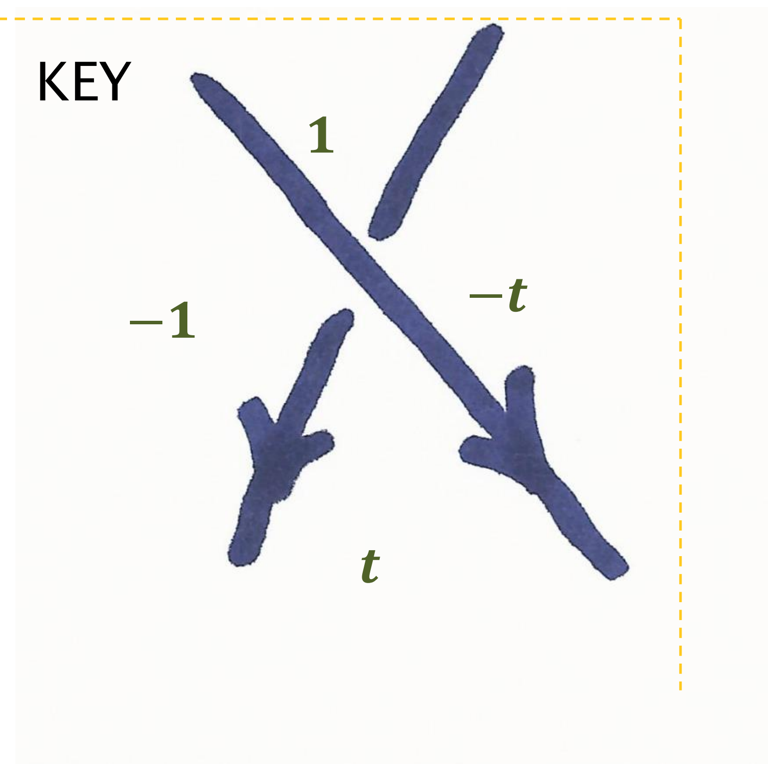

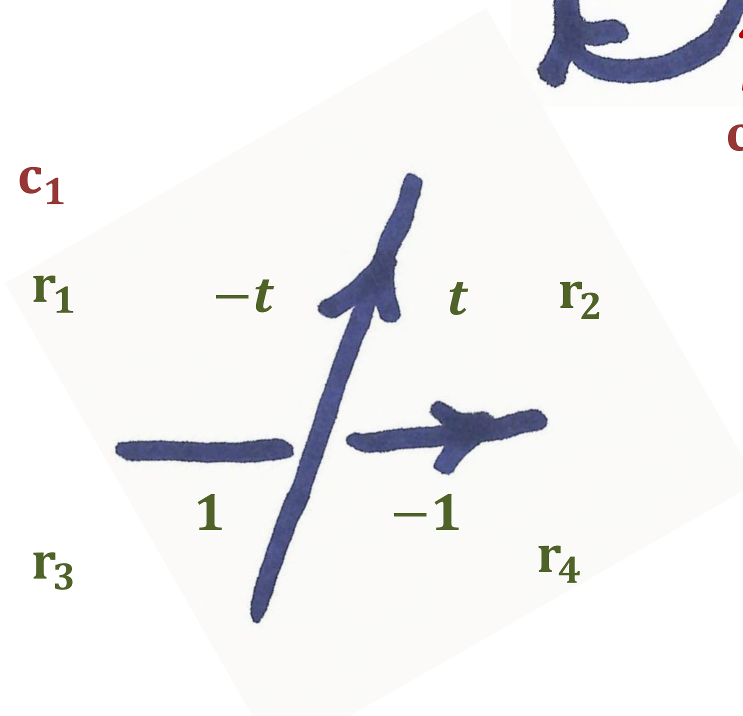

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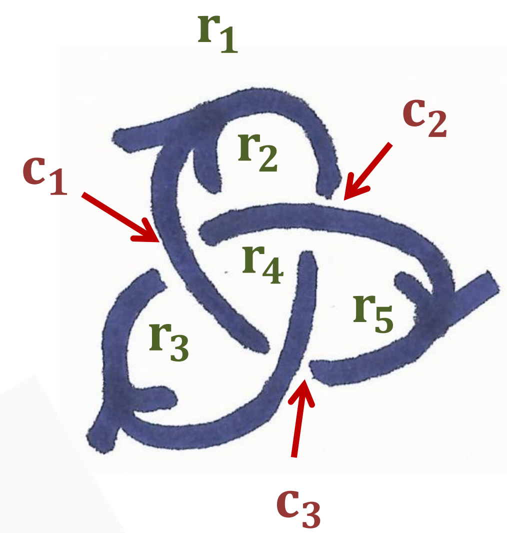
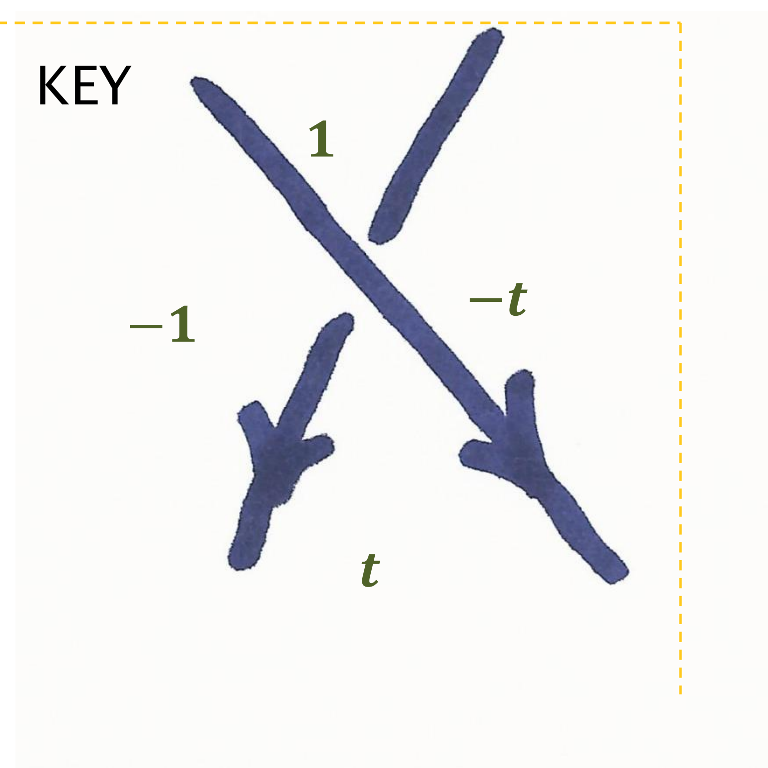
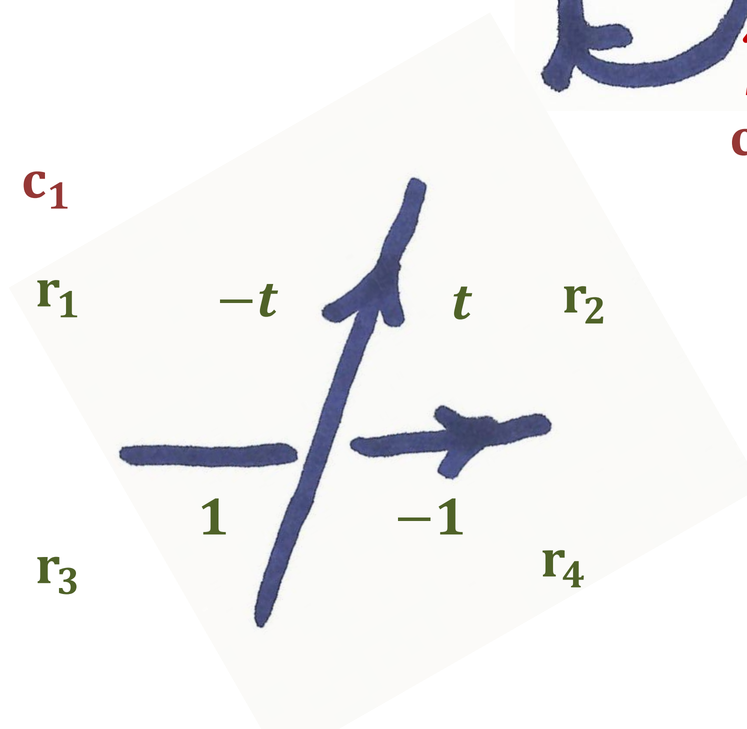
$$M = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{ccccc} r_1 & r_2 & r_3 & r_4 & r_5 \end{array} \\ \begin{pmatrix} & & & & 0 \\ & & & & \\ & & & & \end{pmatrix} \end{array}$$







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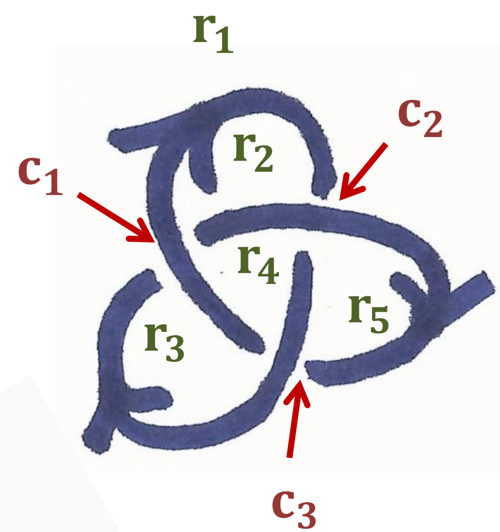
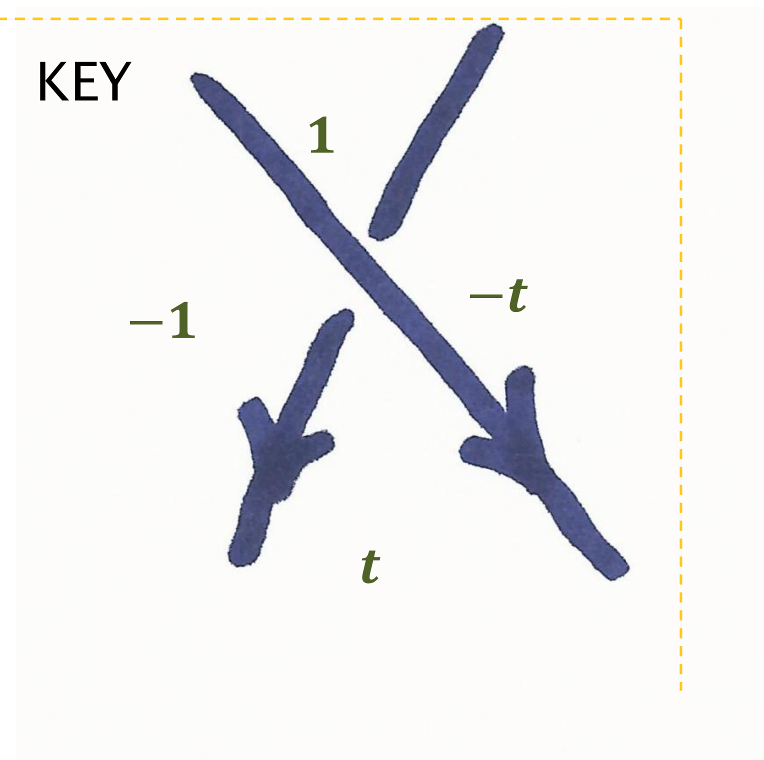
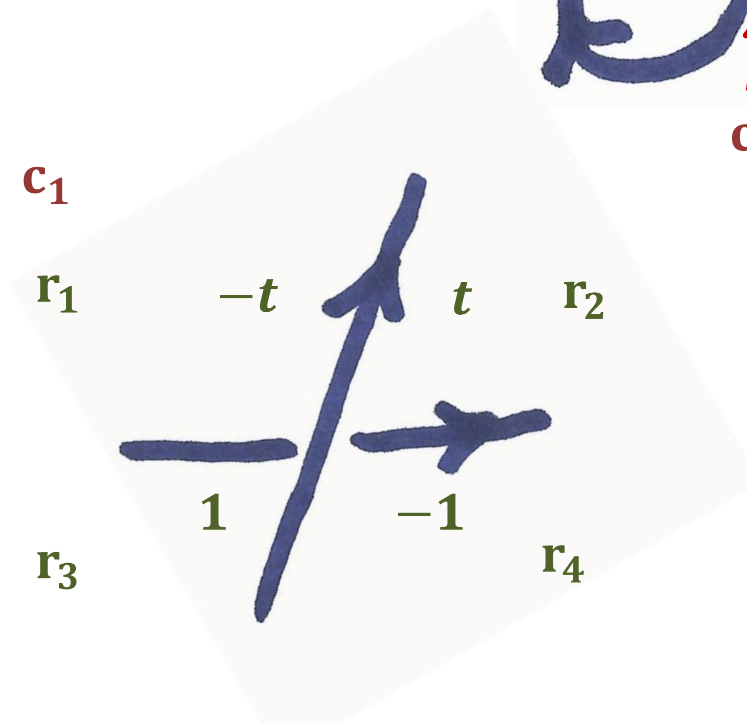


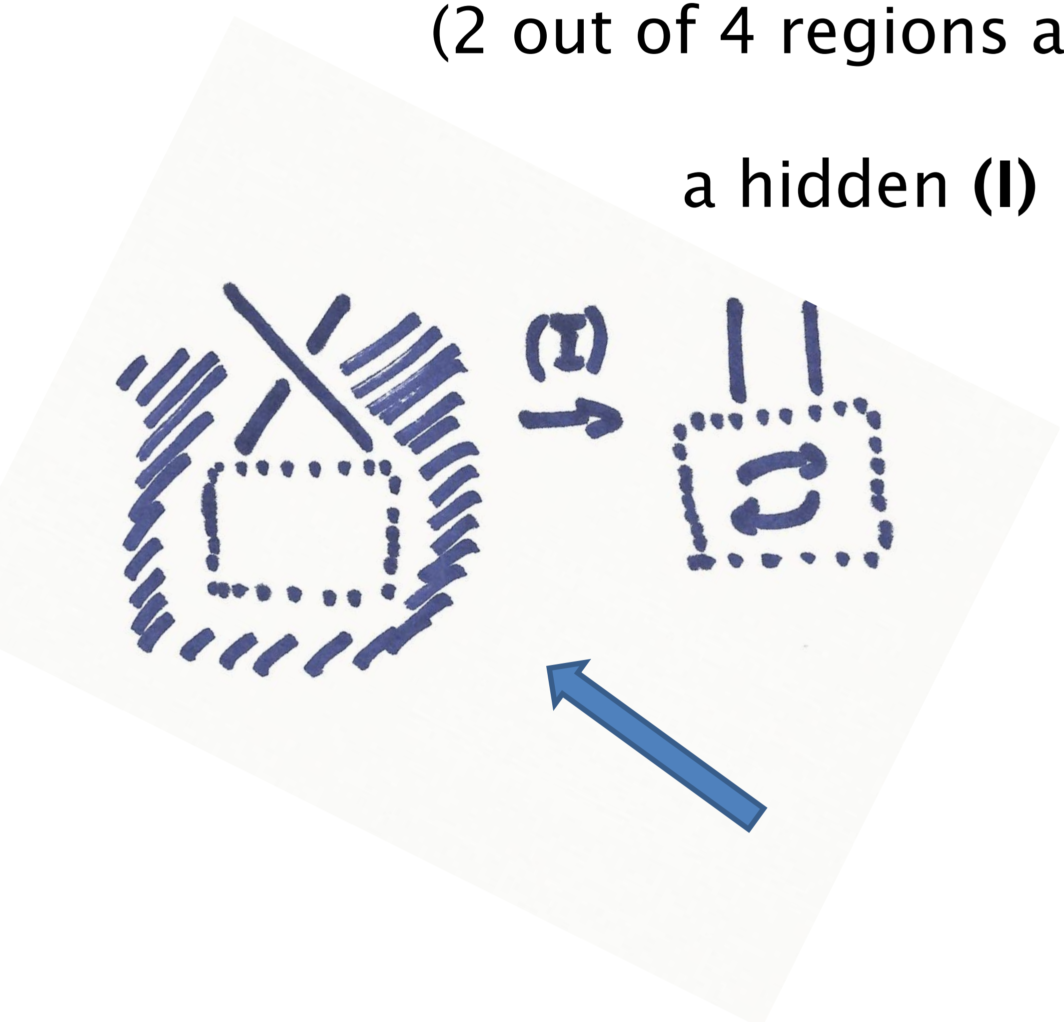




WARNING!

Crossing with 3 different regions
(2 out of 4 regions are connected)

a hidden **(I) move**:



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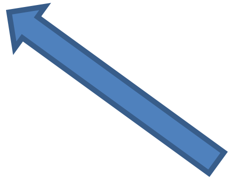

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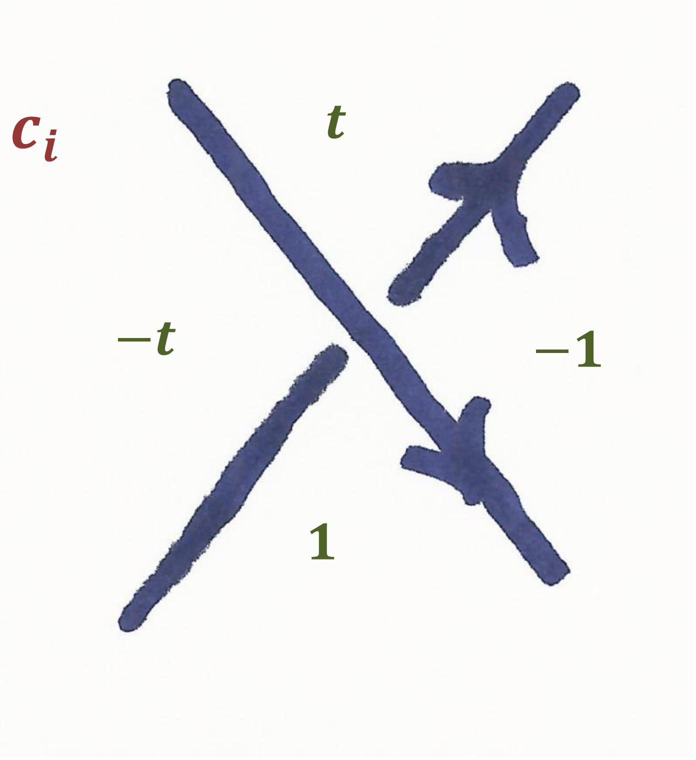

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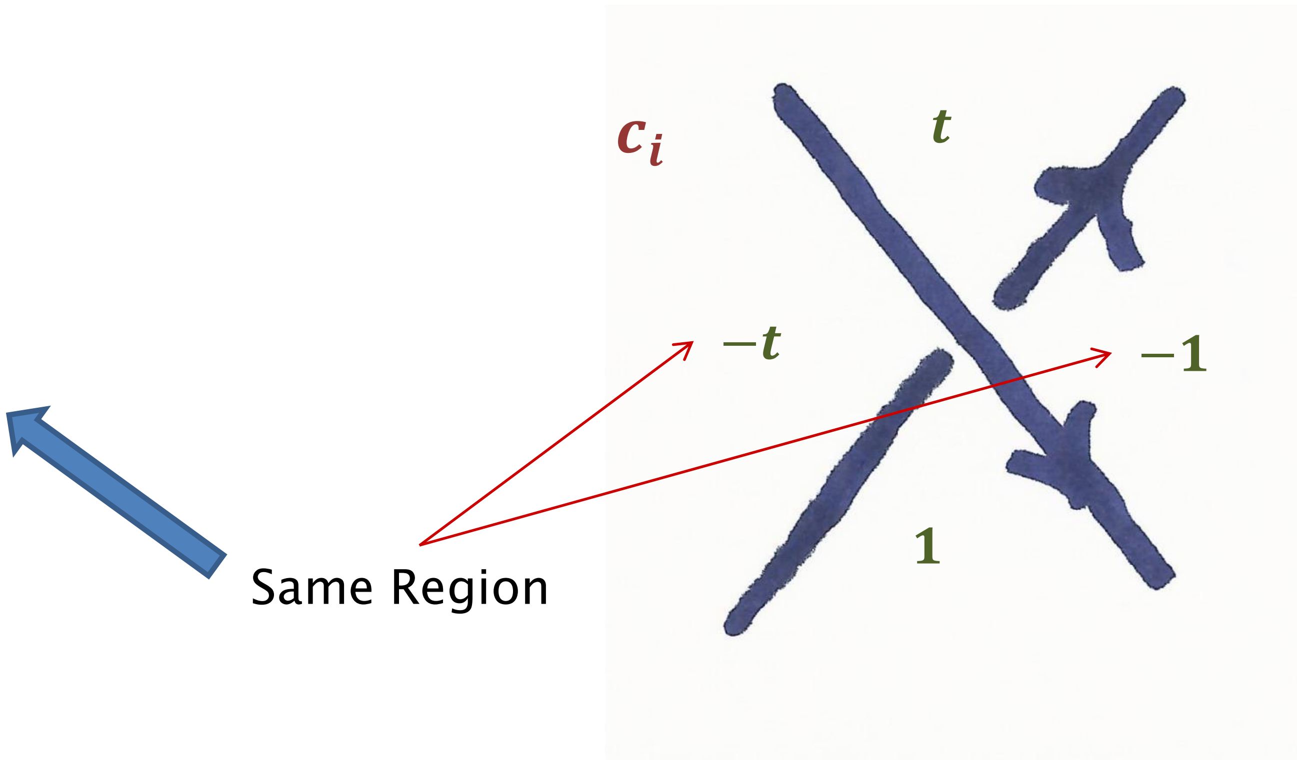

WARNING!

Crossing with 3 different regions
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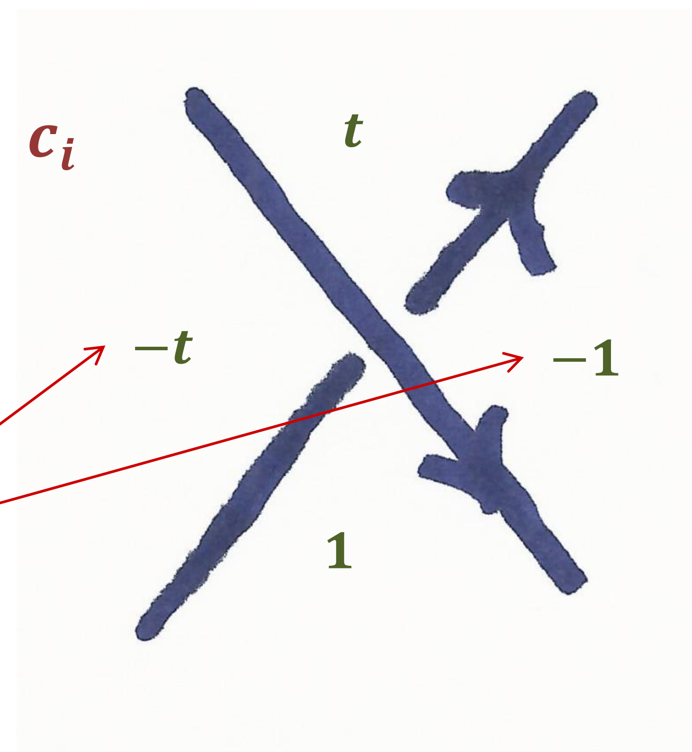
a hidden **(I) move**:

Put the SUM of numbers for that region,
i.e. $-t-1$

Same Region



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- You should have a matrix like this:

$$M = \begin{pmatrix} -t & t & 1 & -1 & 0 \\ -t & 1 & 0 & -1 & t \\ -t & 0 & t & -1 & 1 \end{pmatrix}$$

- Choose two <u>neighboring regions</u>, for example r_4, r_5

- Delete their columns, to get a square matrix

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$$M' = \begin{pmatrix} -t & t & 1 \\ -t & 1 & 0 \\ -t & 0 & t \end{pmatrix}$$

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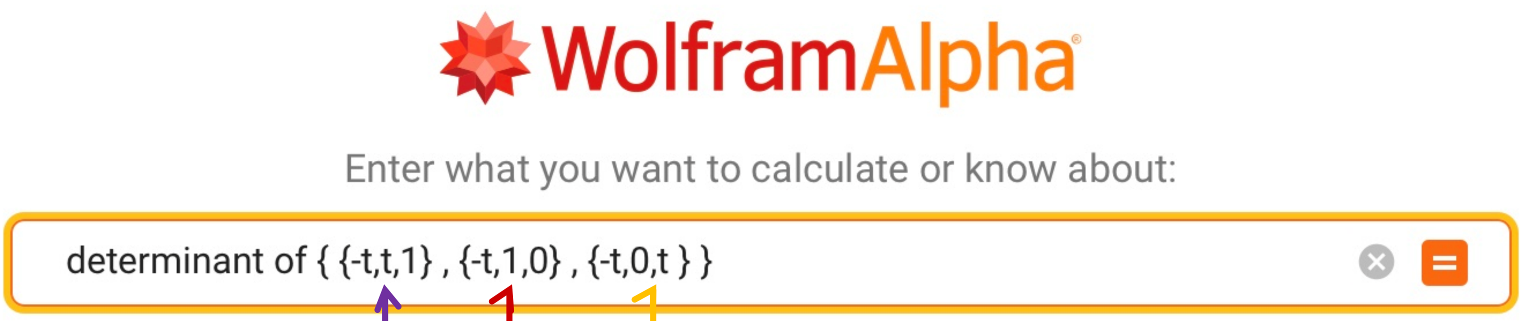

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- Now you should have a polynomial, for example

$$\det(M') = t^3 - t^2 + t = t(t^2 - t + 1)$$

- We made a lot of choice (which regions to delete)

- In my example

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  $\Rightarrow \quad -7t^3 - 3t + \mathbf{5}$

  - Make highest power of t , have a **positive** coefficient

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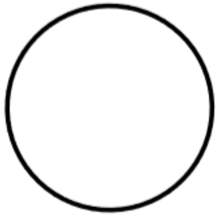
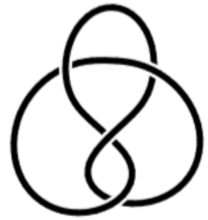
(Alexanders's Theorem, 1928)
The procedure described above gives Knot Invariants

A table of prime knots of 7 or fewer crossings with their Alexander polynomials



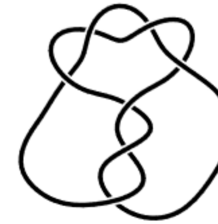
$\mathbf{0_1}$
1

$\mathbf{3_1}$
$1 - t + t^2$

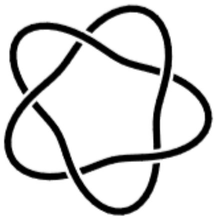


$\mathbf{4_1}$
$1 - 3t + t^2$

$\mathbf{7_2}$
$3 - 5t + 3t^2$

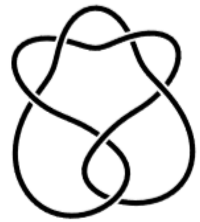


$\mathbf{7_3}$
$2 - 3t + 3t^2 - 3t^3 + 2t^4$

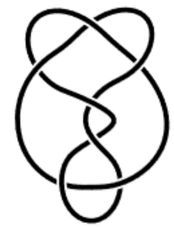


$\mathbf{5_1}$
$1 - t + t^2 - t^3 + t^4$

$\mathbf{5_2}$
$2 - 3t + 2t^2$

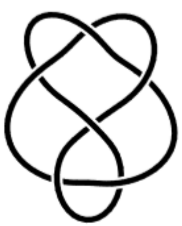


$\mathbf{6_1}$
$2 - 5t + 2t^2$

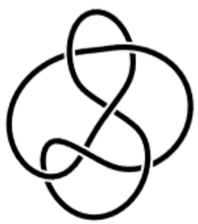


$\mathbf{7_5}$
$2 - 4t + 5t^2 - 4t^3 + 2t^4$

$\mathbf{7_6}$
$1 - 5t + 7t^2 - 5t^3 + t^4$



$\mathbf{6_2}$
$1 - 3t + 3t^2 - 3t^3 + t^4$



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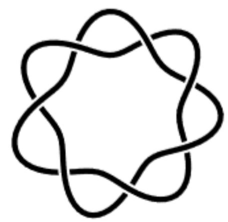
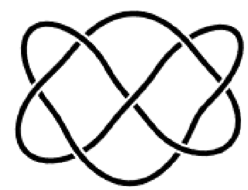


$\mathbf{7_1}$
$1 - t + t^2 - t^3 + t^4 - t^5 + t^6$



$\mathbf{7_4}$
$4 - 7t + 4t^3$

$\mathbf{7_7}$
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Assumptions for
Alexander Polynomial

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- First Choose an *orientation*

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- First Choose an *orientation*
- ***(Euler's Theorem)*** *A knot diagram with n crossings, divides plane into n+2 regions*
- Name your regions ($r_1, r_2, \cdots, r_{n+2}$) and crossings ($c_1, c_2, \cdots, c_n$)
- Draw a Matrix with *n* rows and *n+2* columns

$$\begin{array}{c c} & \begin{array}{ccccc} r_1 & r_2 & r_3 & r_4 & r_5 \end{array} \\ M = \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} & \left(\begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \end{array} \right) \end{array}$$

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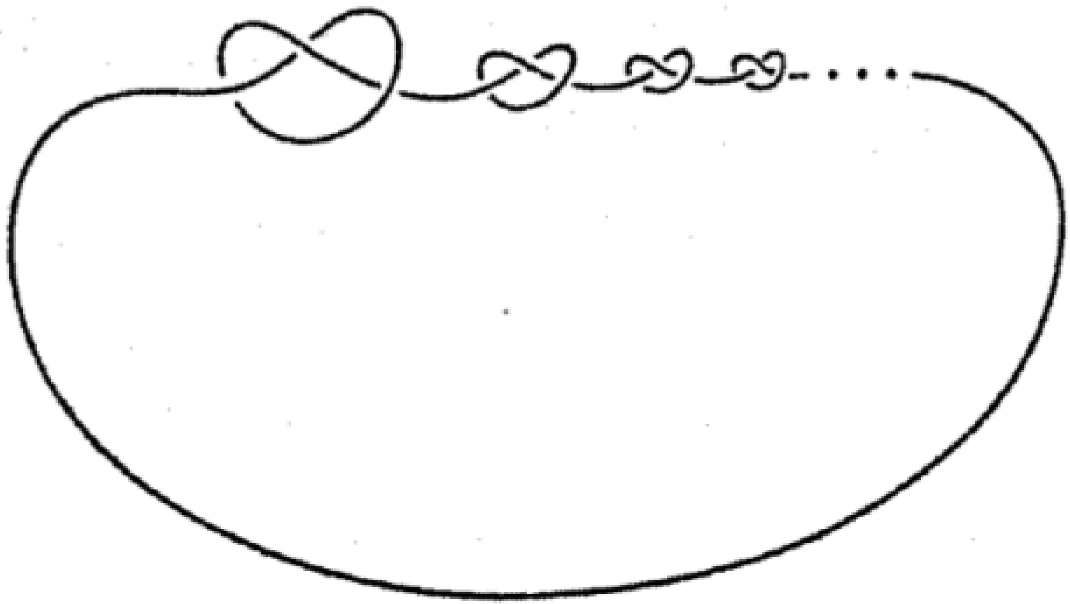

Invariants:

Assign a “number” to each Knot called it’s invariant so that

- Equivalent knots get the same number

Then, if two knots get different numbers, then they’re not equivalent.

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- Tells knots of $n < 9$ crossings apart

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Need to check if Alexander Polynomial doesn't change after a Reidmeister move!

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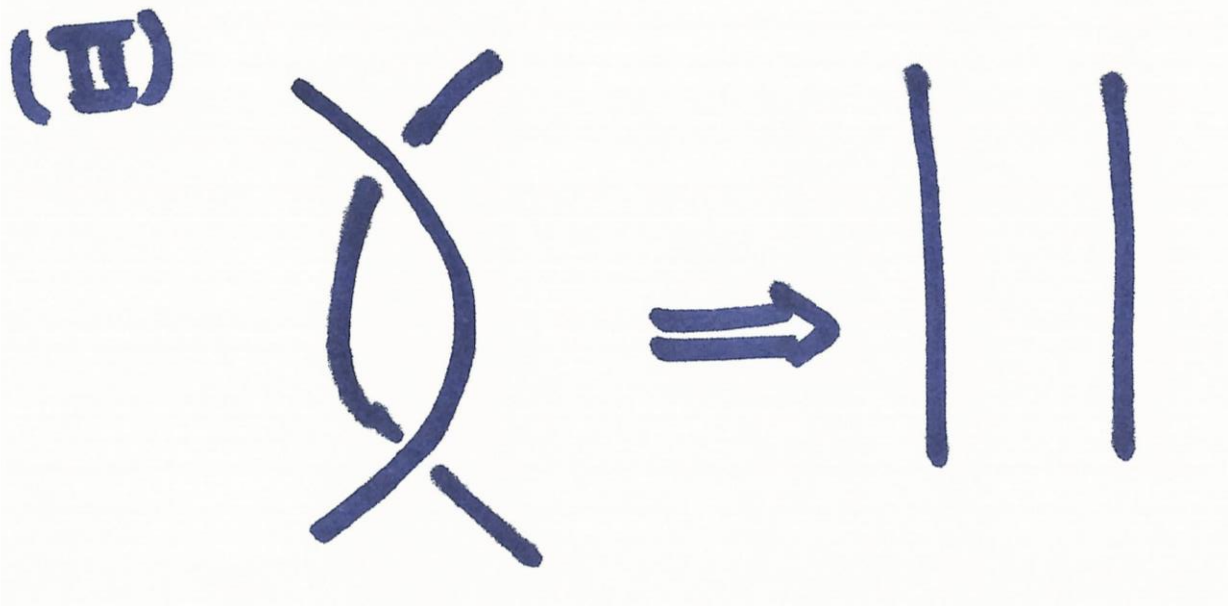

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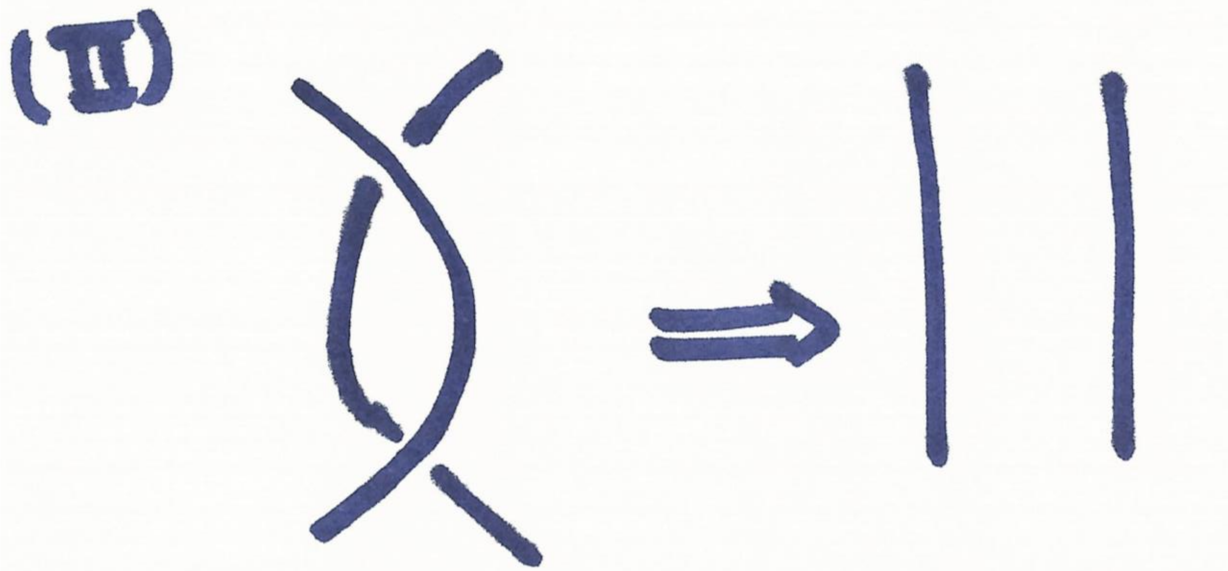

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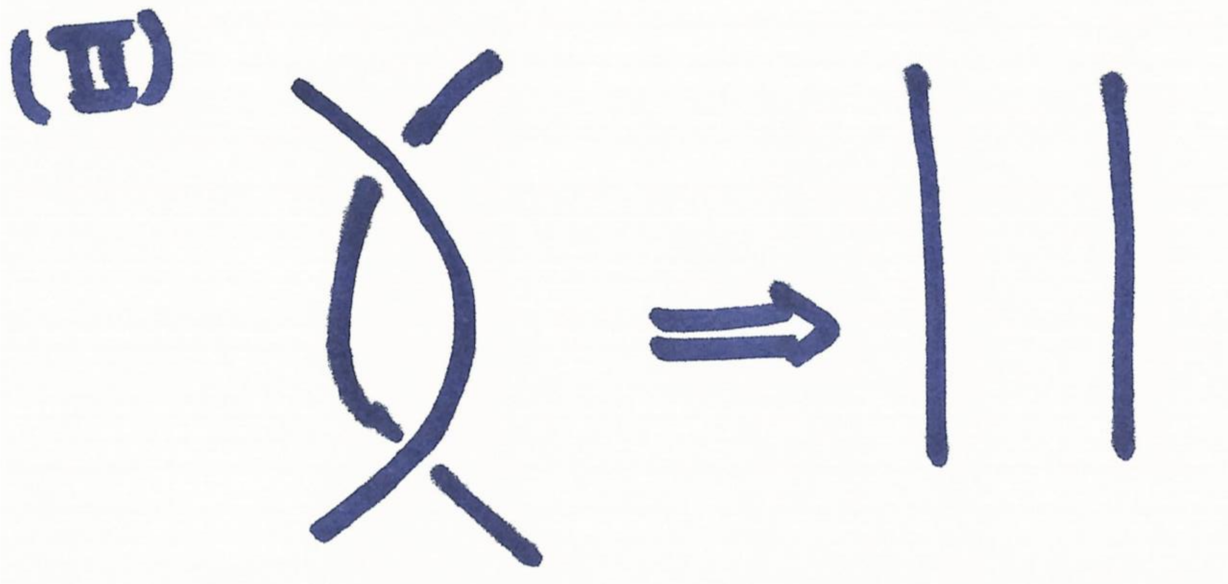

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Tricolourability

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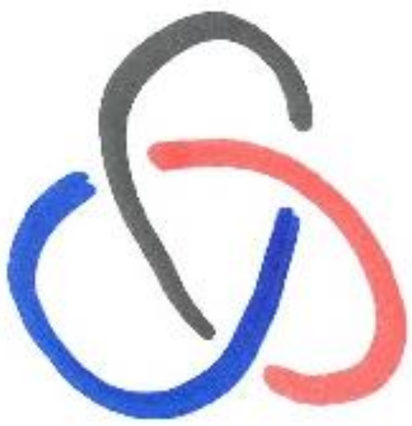

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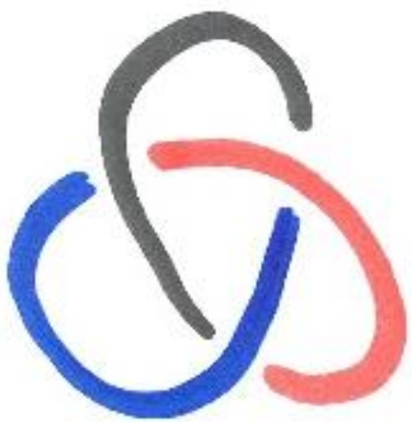

We say a knot is Tricolourable if we can colour its arcs with ***<u>3 colours</u>*** and

- At every crossing, either all three colours or only one colour is used.
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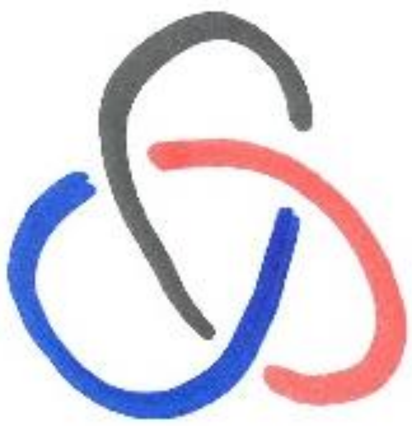

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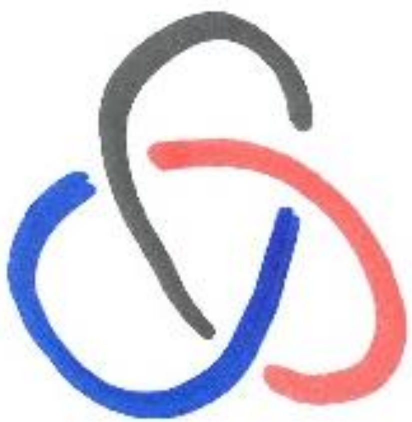

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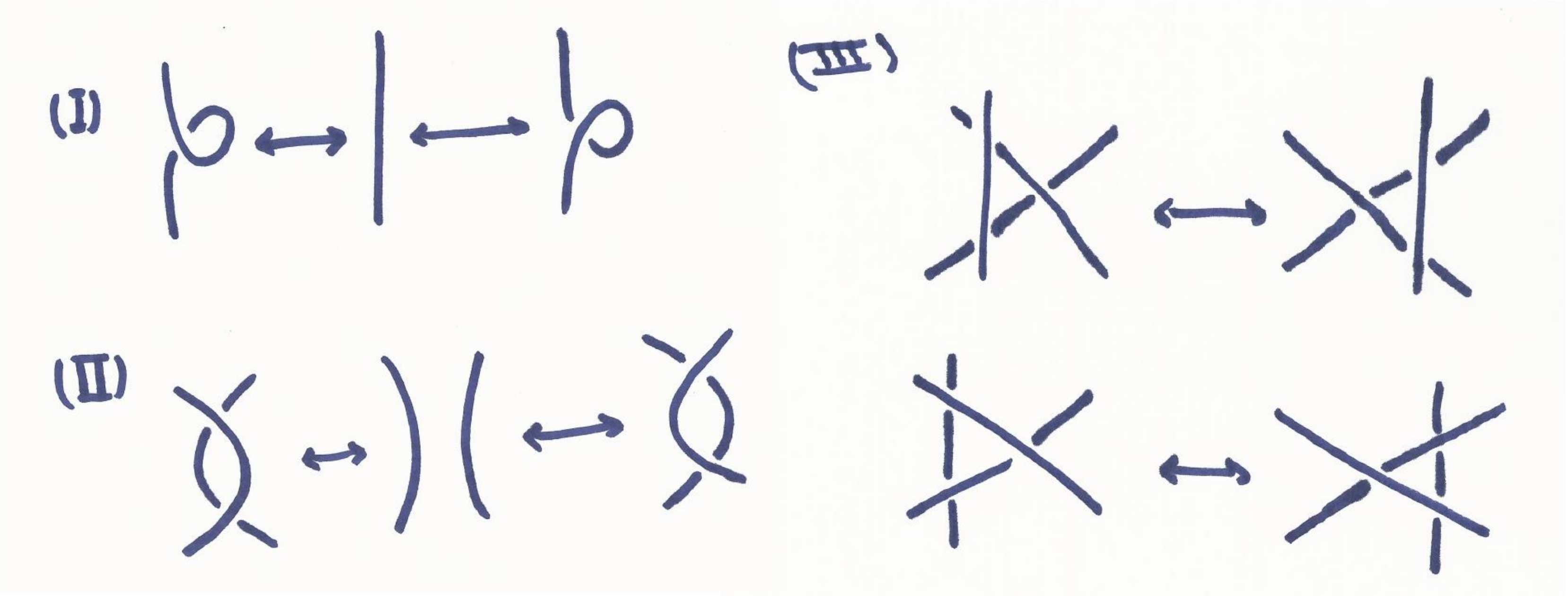

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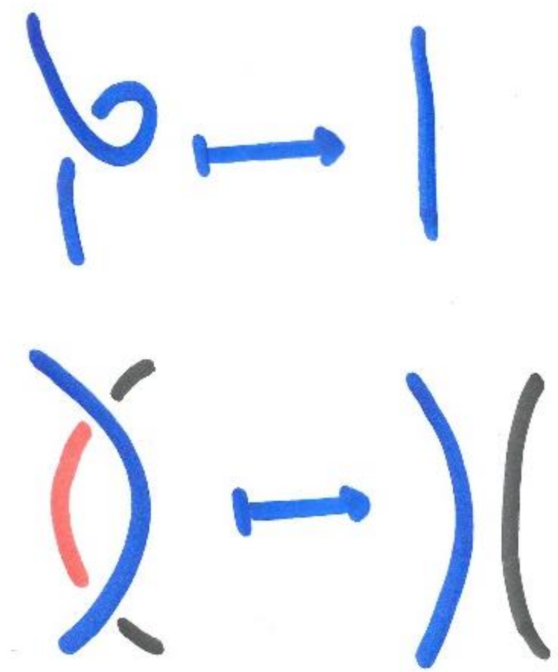

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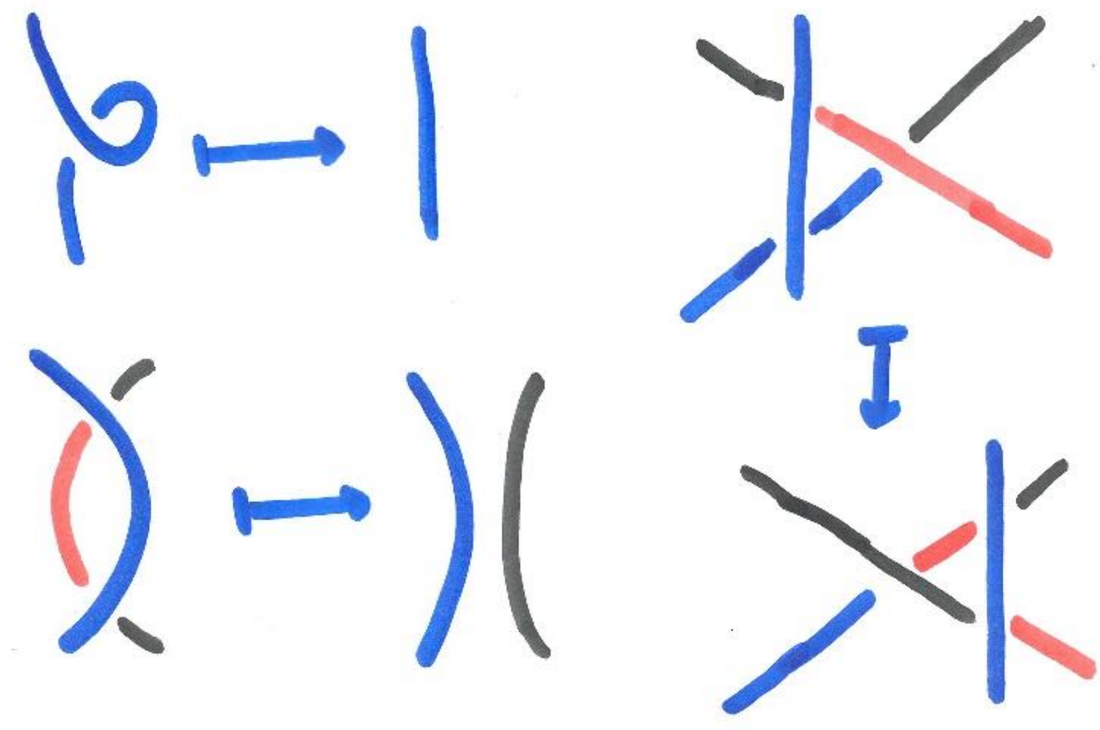

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- Fix a point in space (outside your knot)
- Take the Set G, of all topological directed loops starting and ending at your fixed point!

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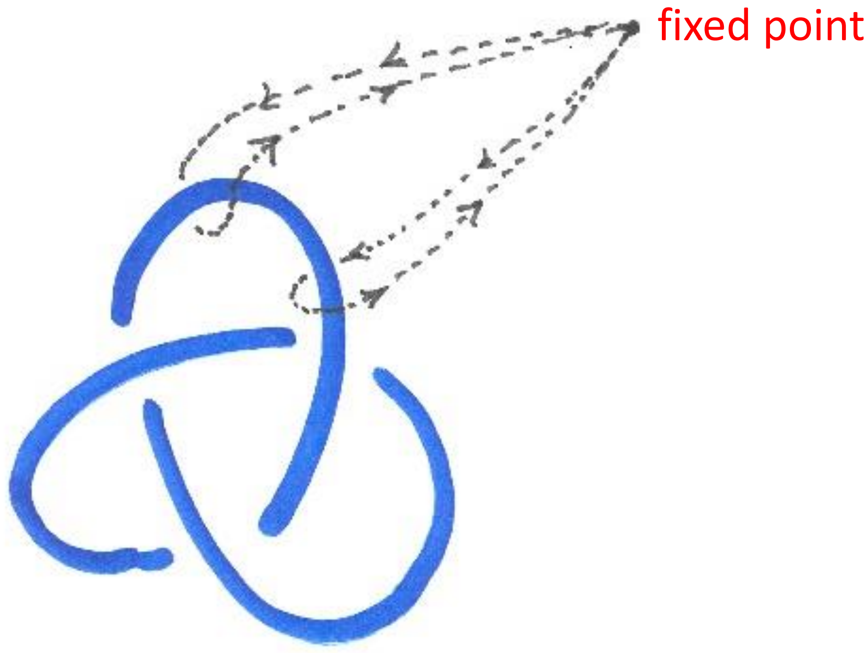

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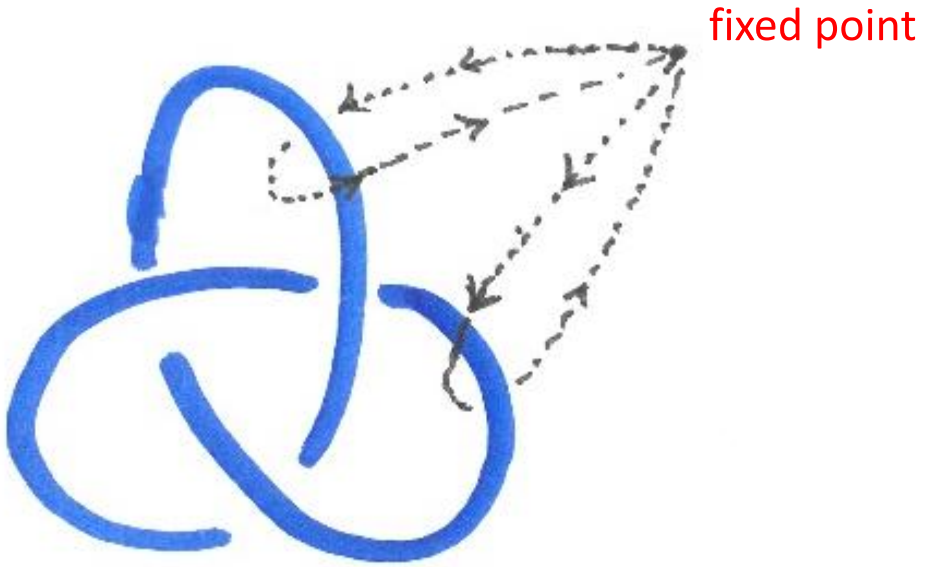

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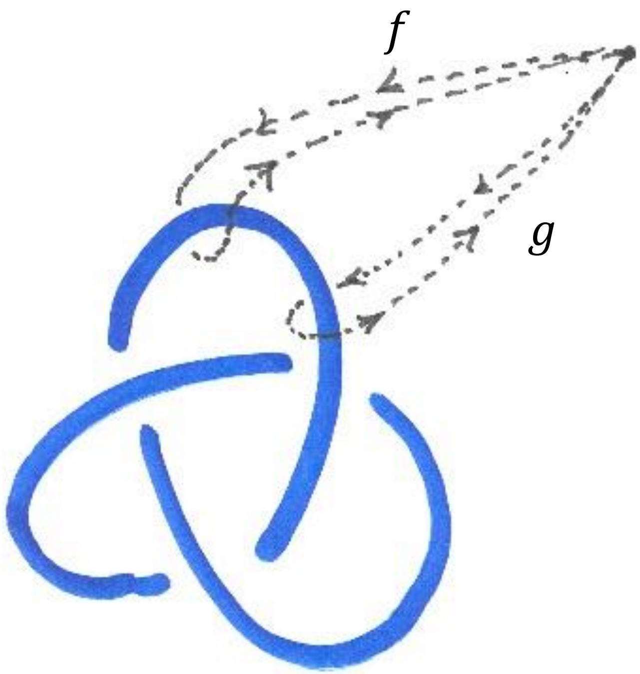

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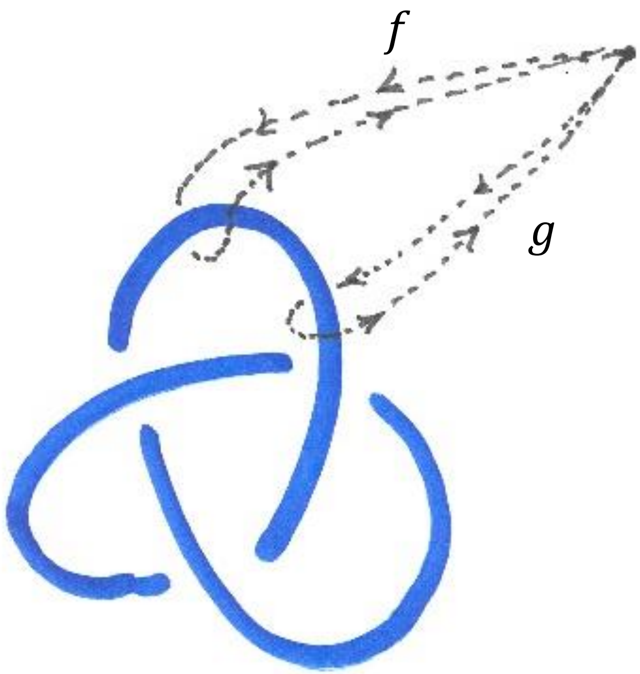

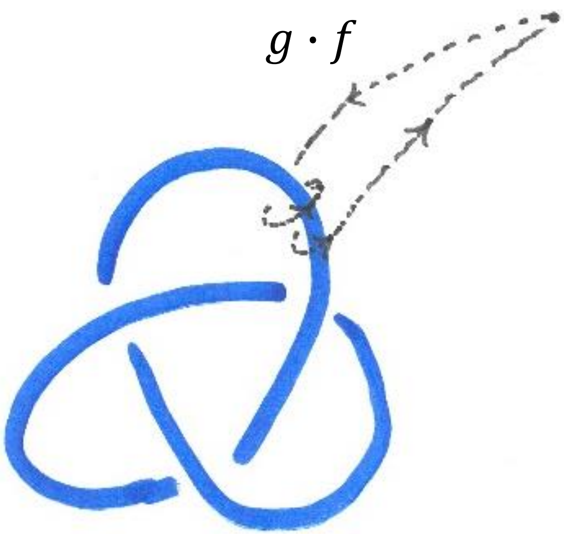

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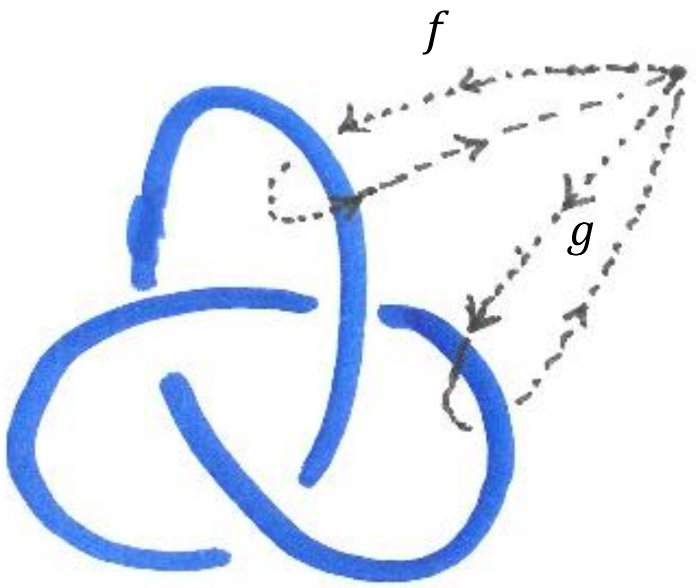

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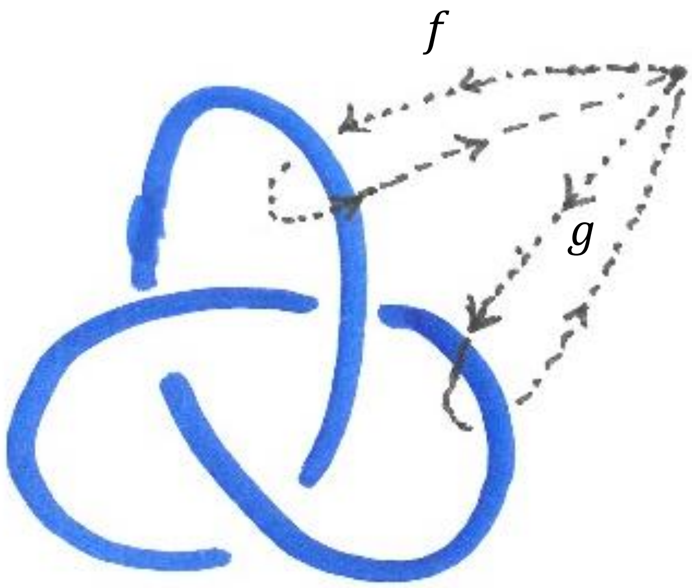

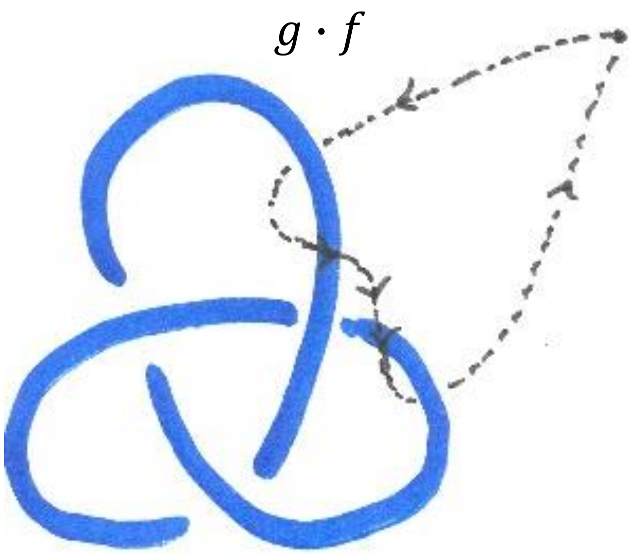

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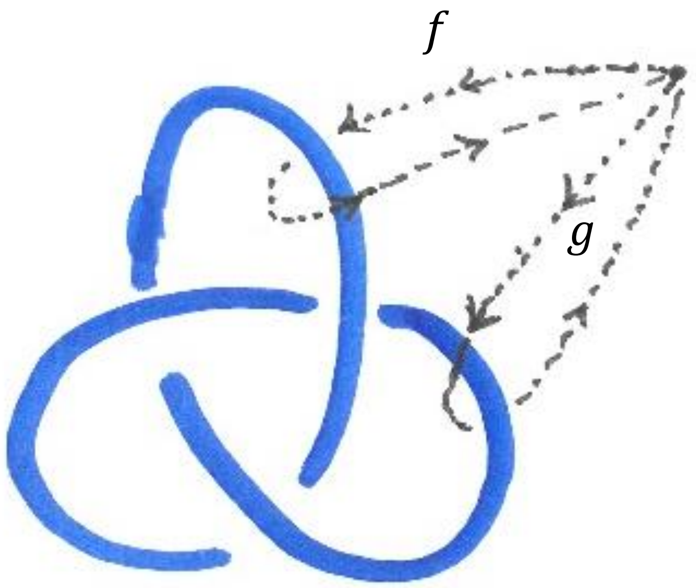

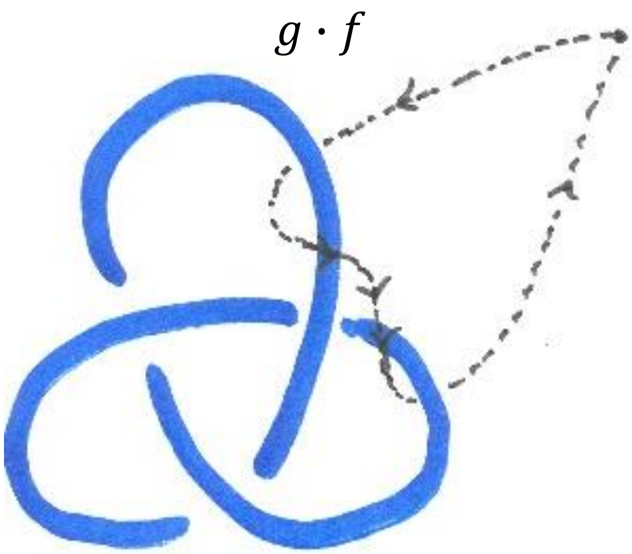

(**Associativity**) Order doesn't matter $(h \cdot g) \cdot f = h \cdot (g \cdot f)$

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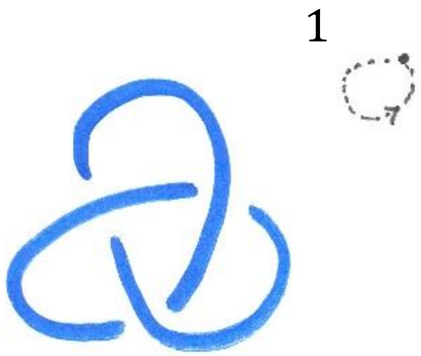

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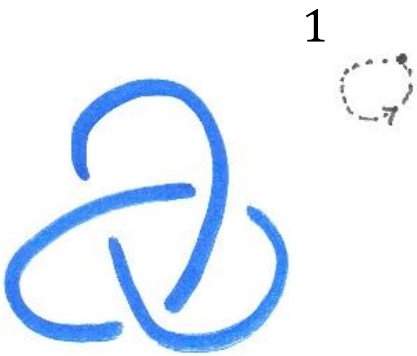

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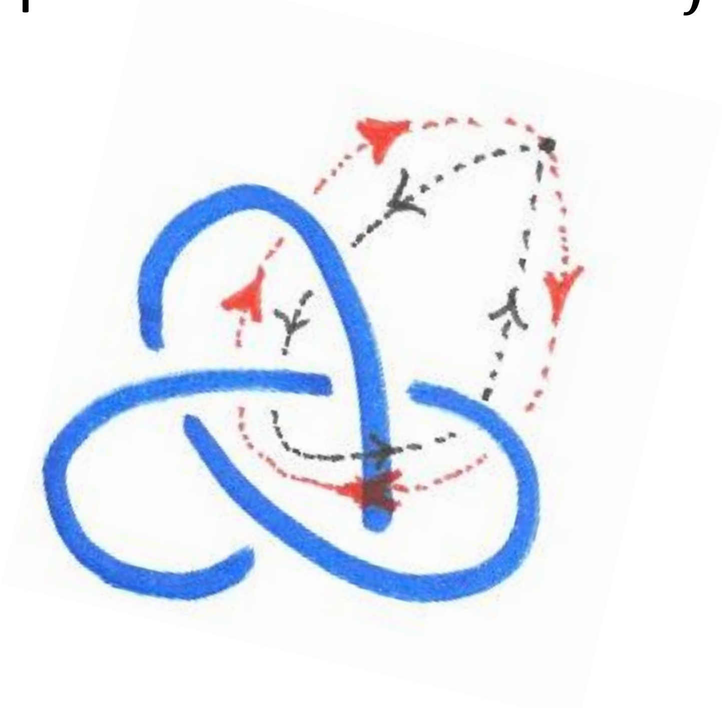

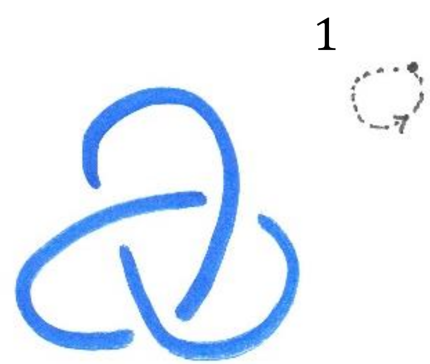

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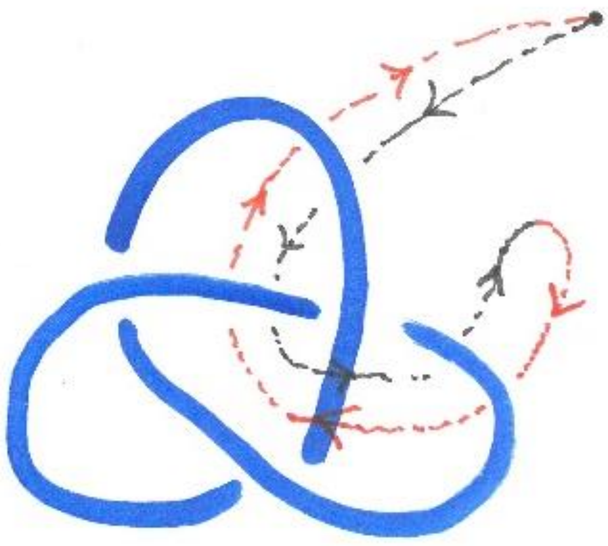

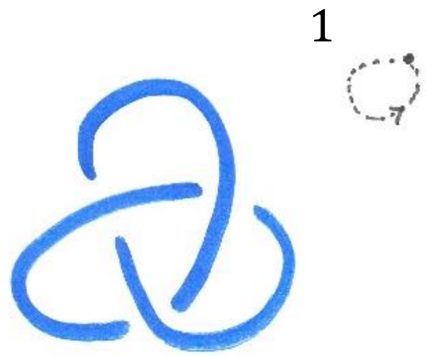

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Formally: a **Group** is

- A set G
- An operation $\cdot : G \times G \to G$
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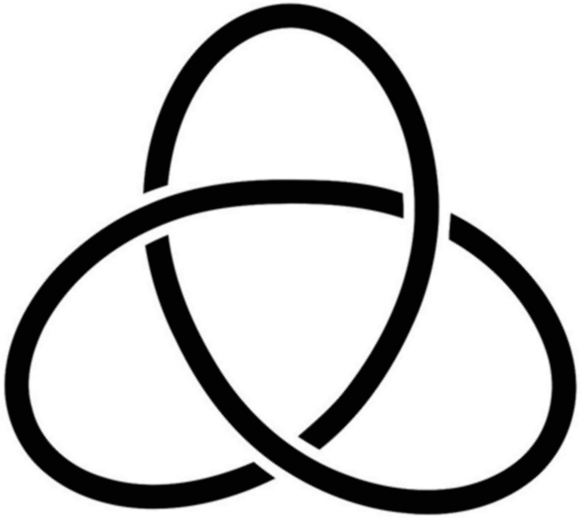

Fundamental Group

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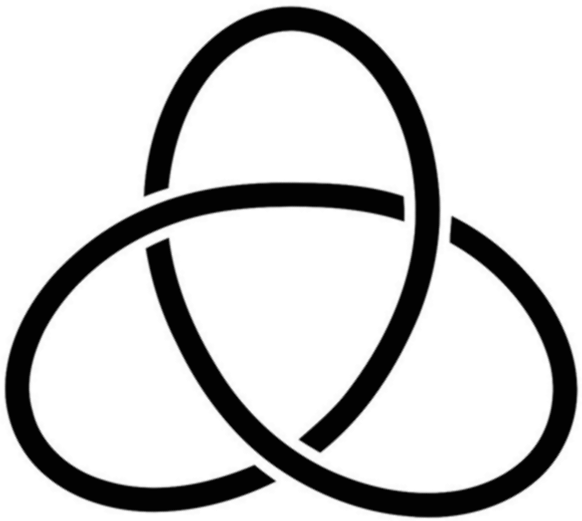
- Relations corresponding to each crossing (in the diagram)



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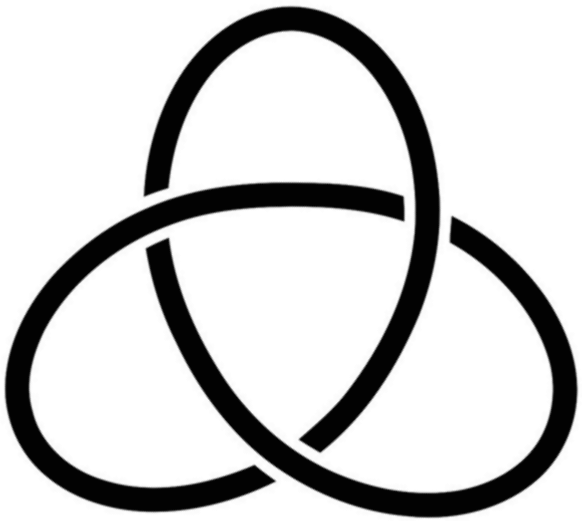

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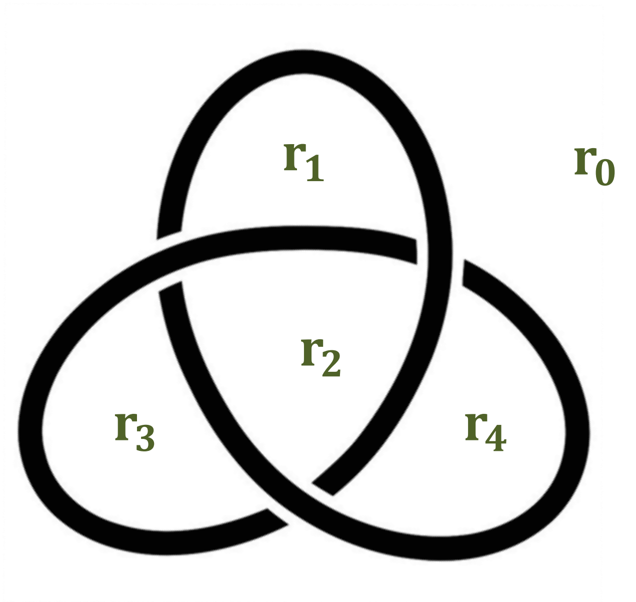

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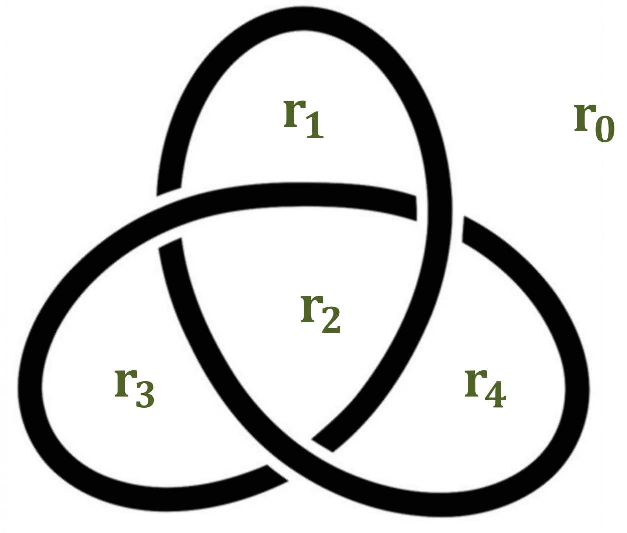

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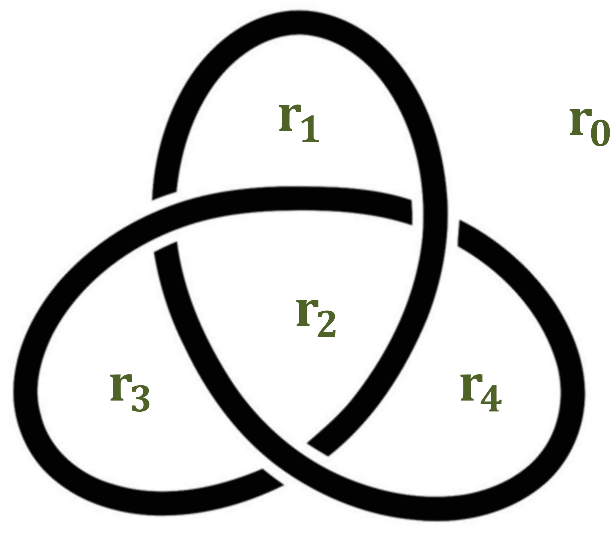

- Group has generators r_0, r_1, r_2, r_3, r_4

And Relations :

$$r_0 = 1$$
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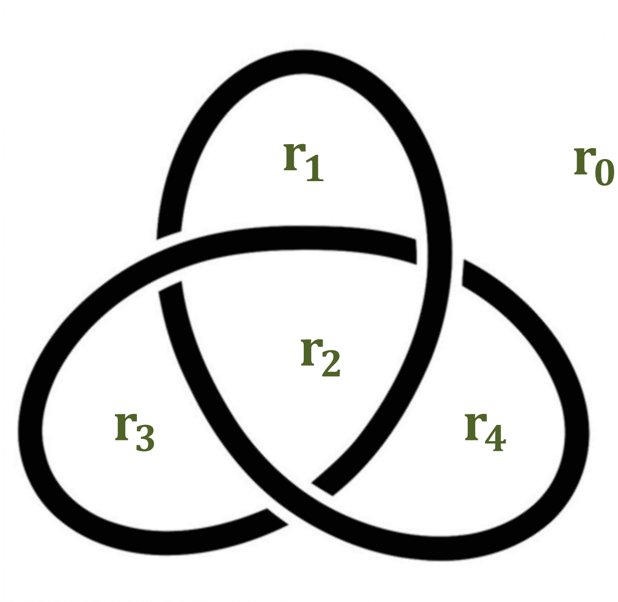

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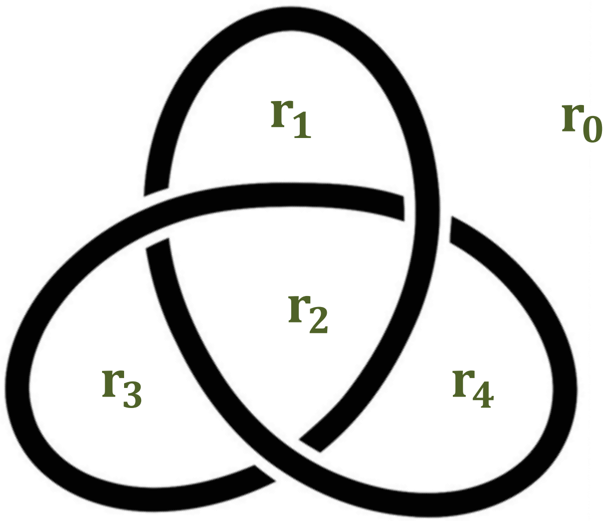

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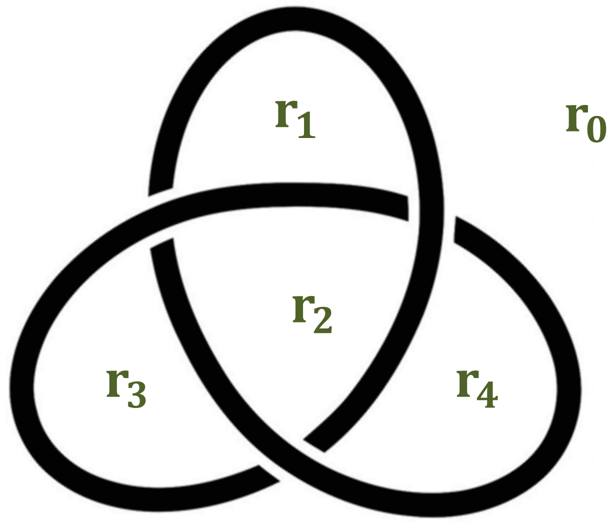

- Group has generators r_0, r_1, r_2, r_3, r_4

And Relations :

$$r_0 = 1$$
$$r_1 = r_2 \cdot r_4^{-1}$$
$$r_3 = r_1 \cdot r_2^{-1}$$
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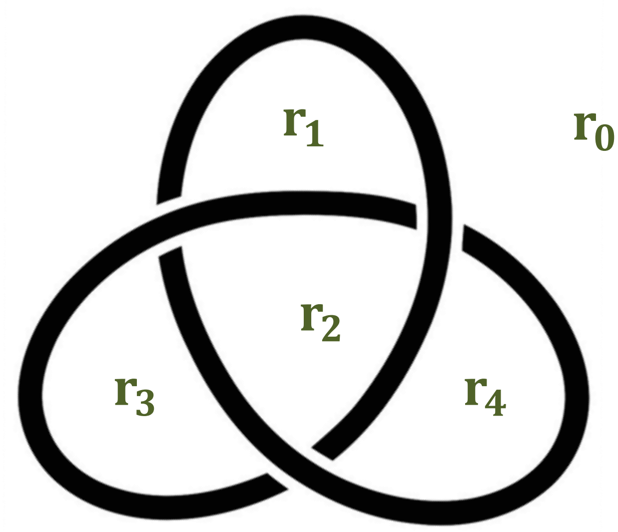

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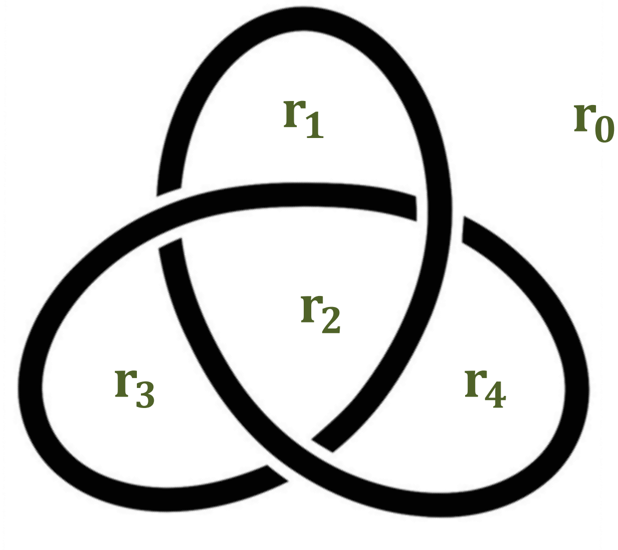

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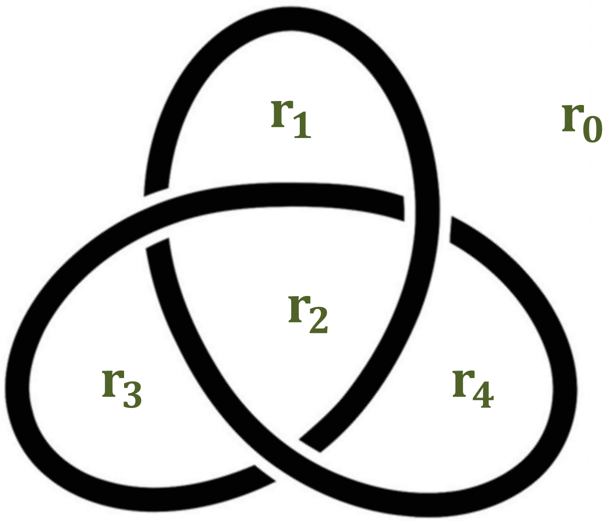

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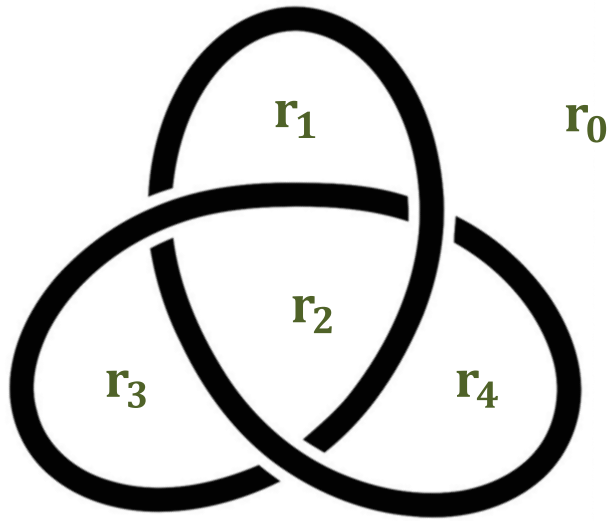

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Theorem. The (isomorphism type) fundamental group of a knot, is a knot invariant.

- *Good mathematics should connect with other good mathematics!*

Knots

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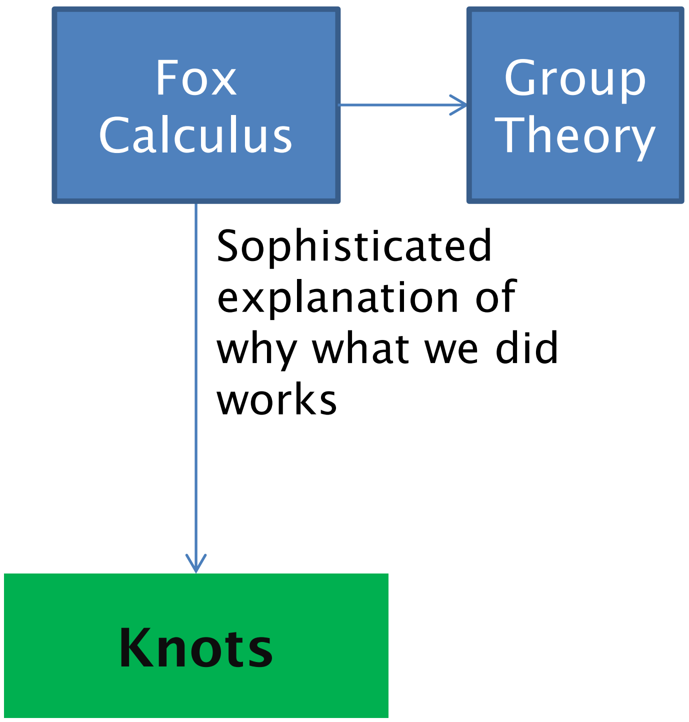

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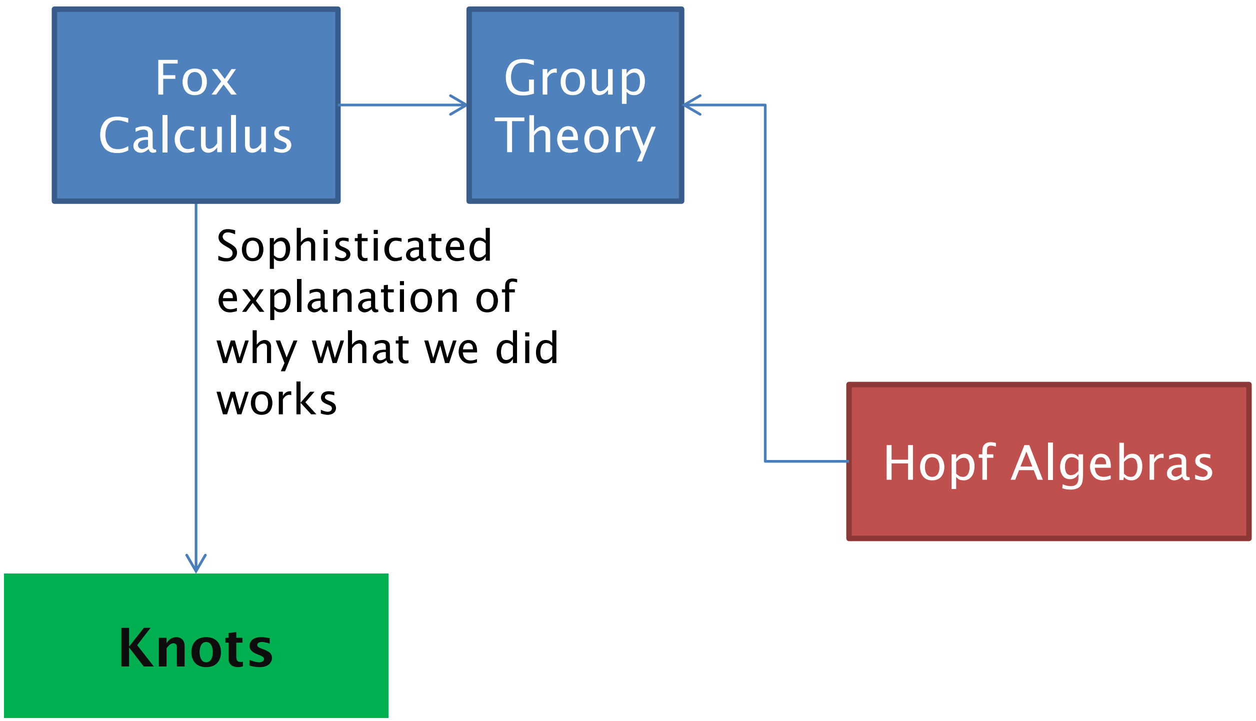

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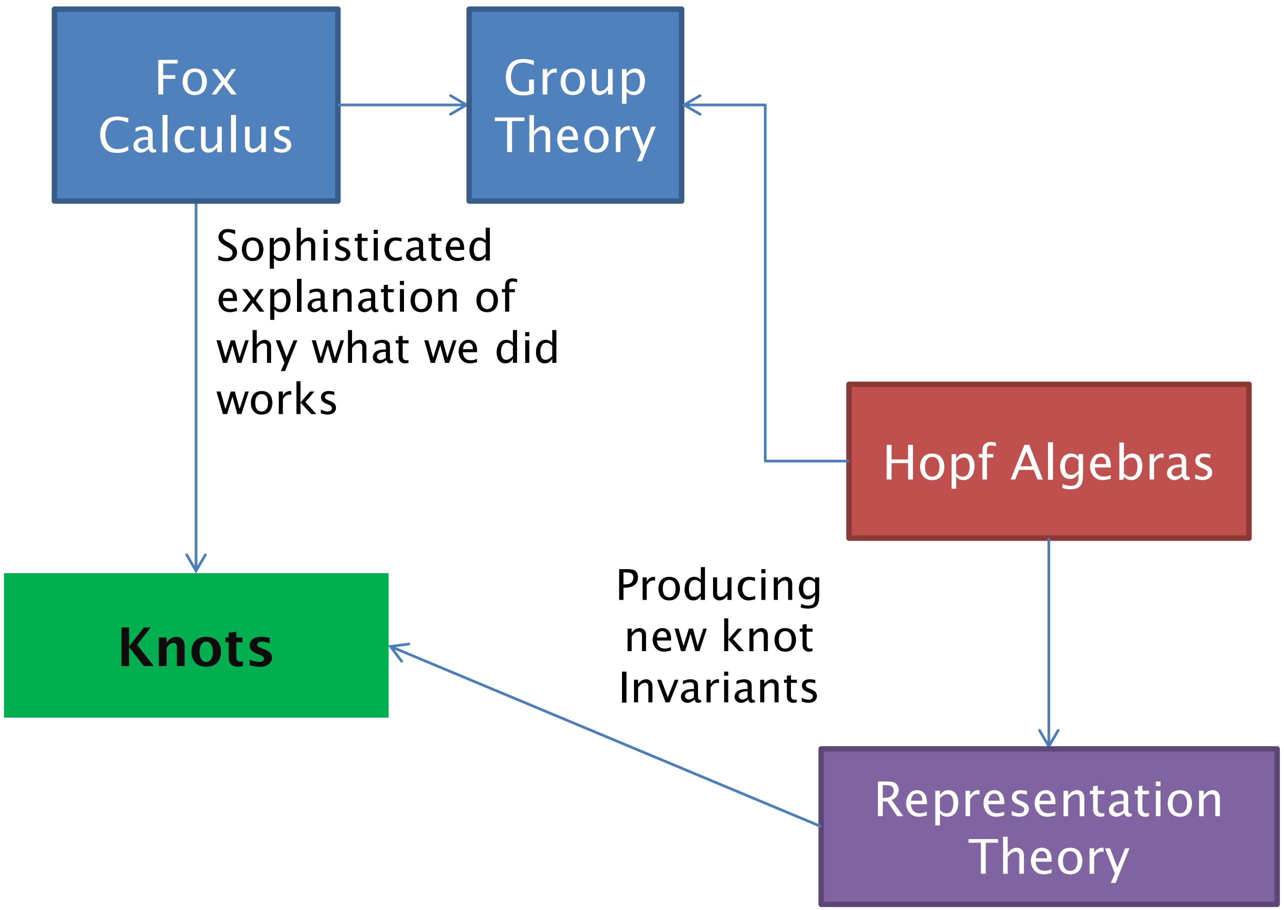

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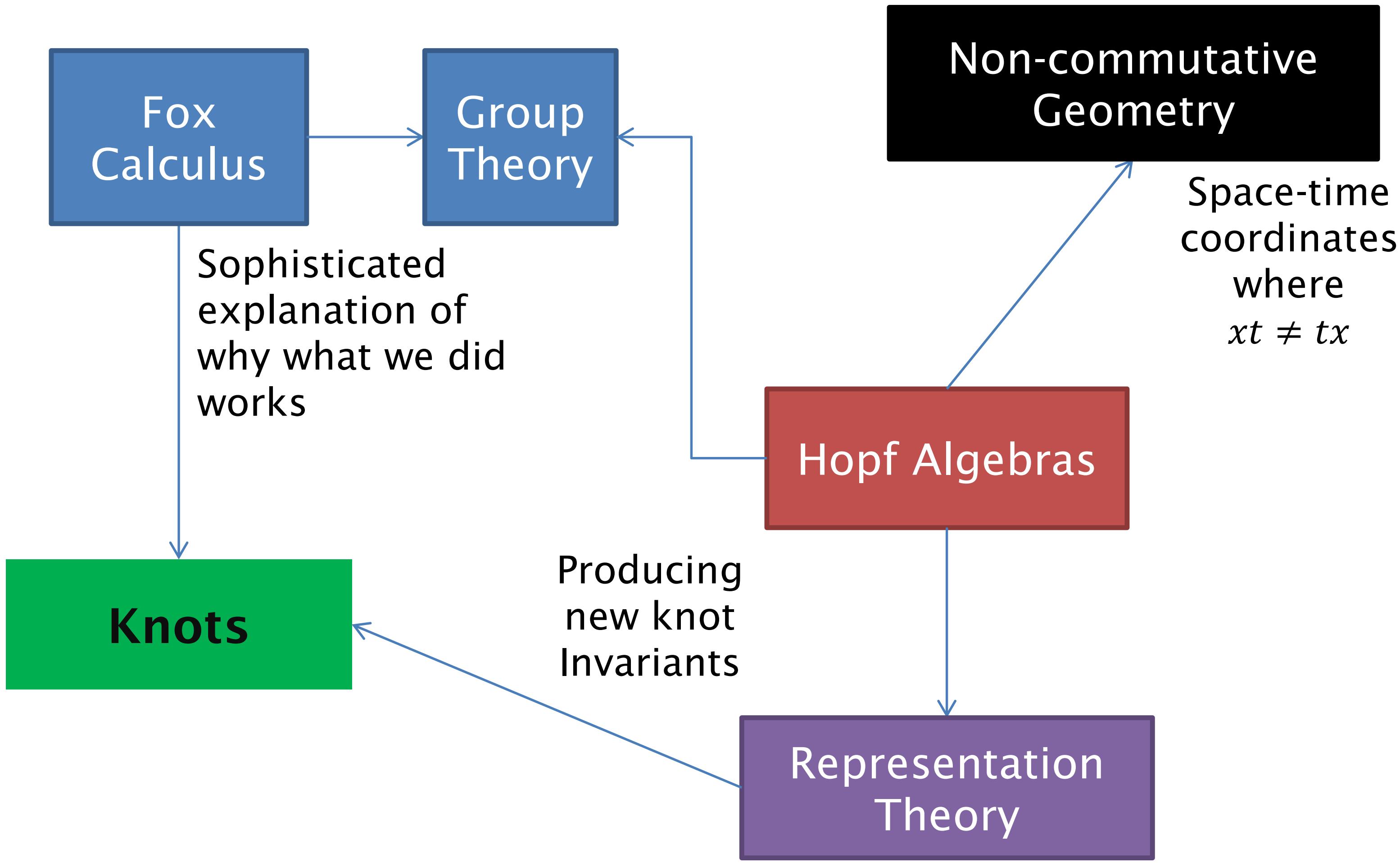

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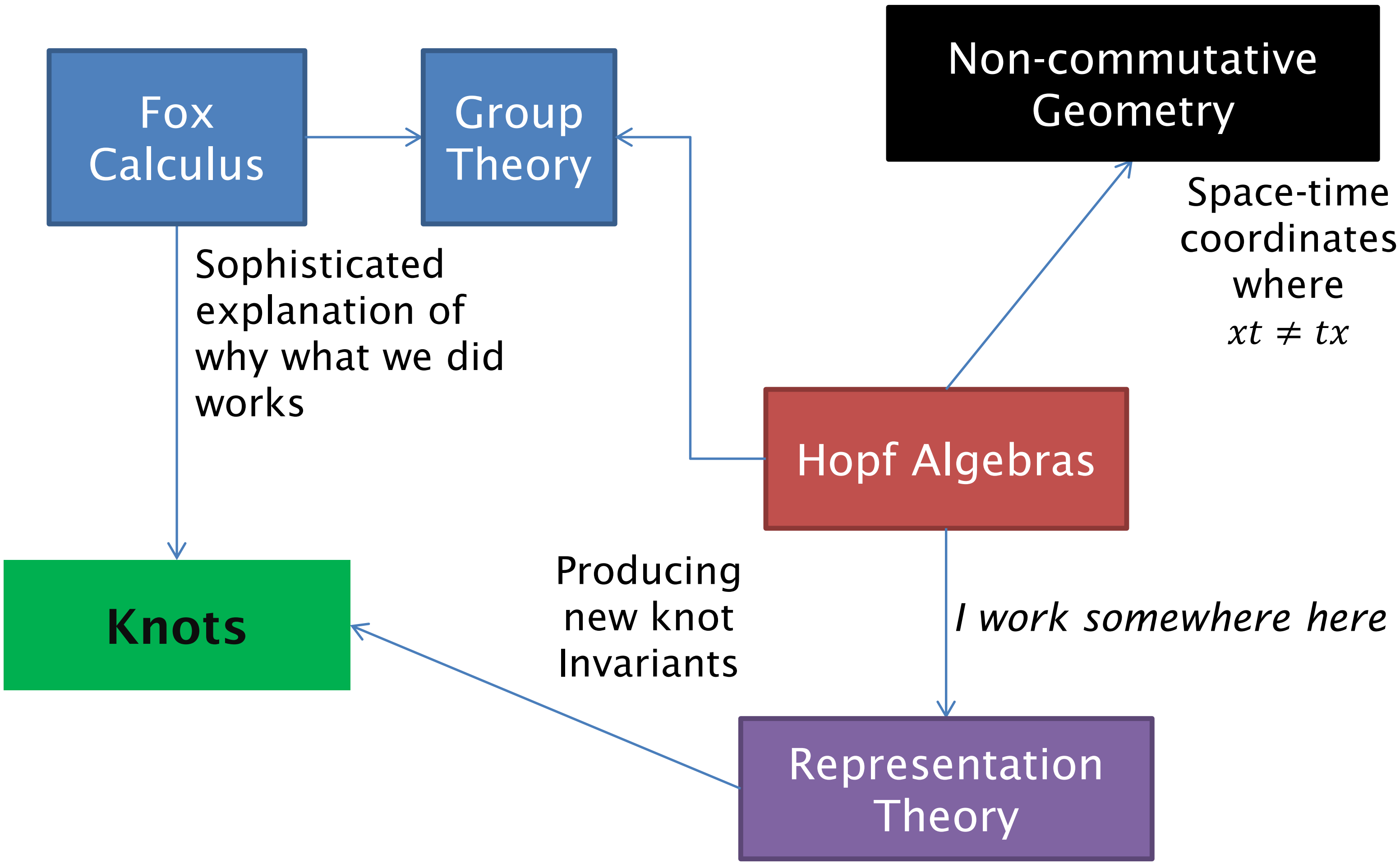
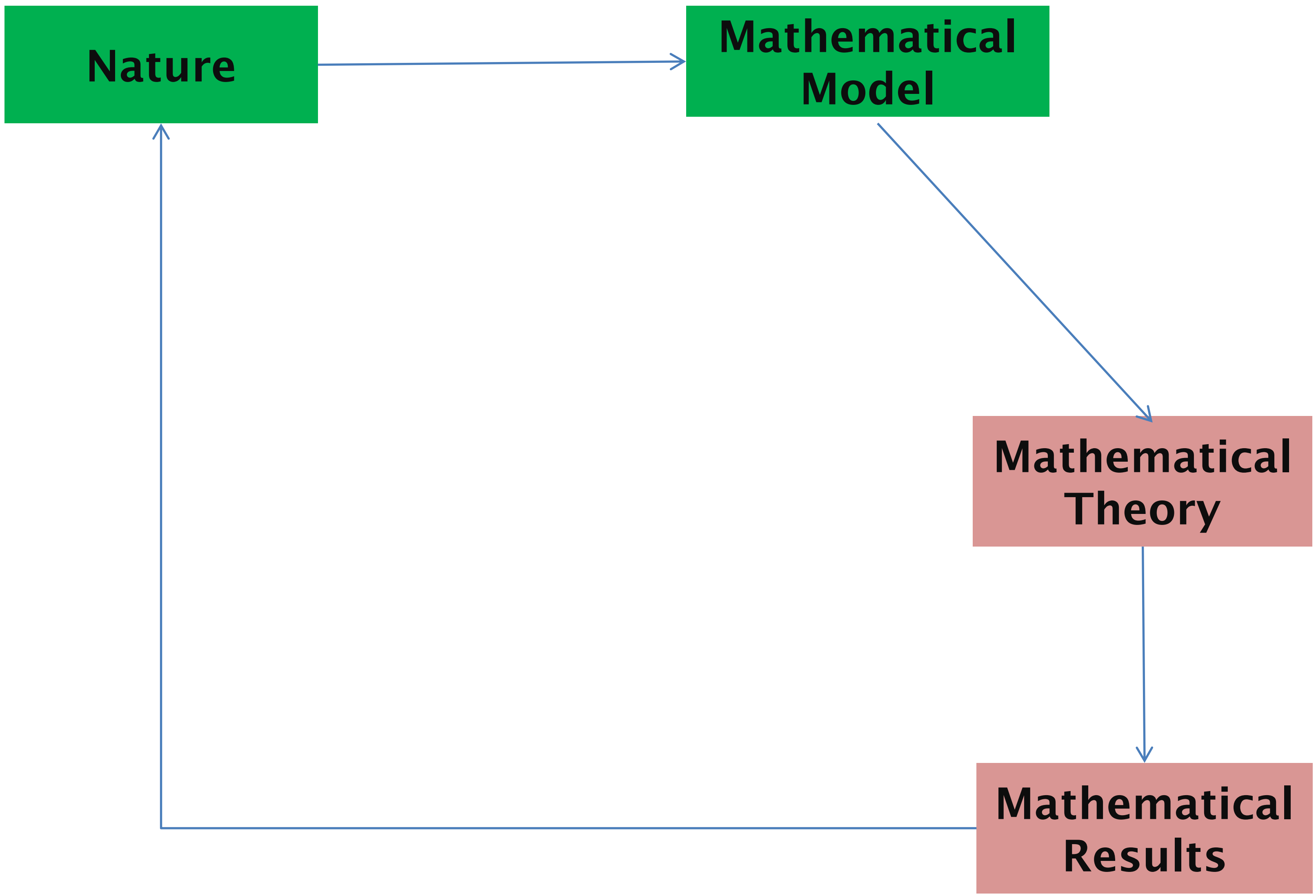
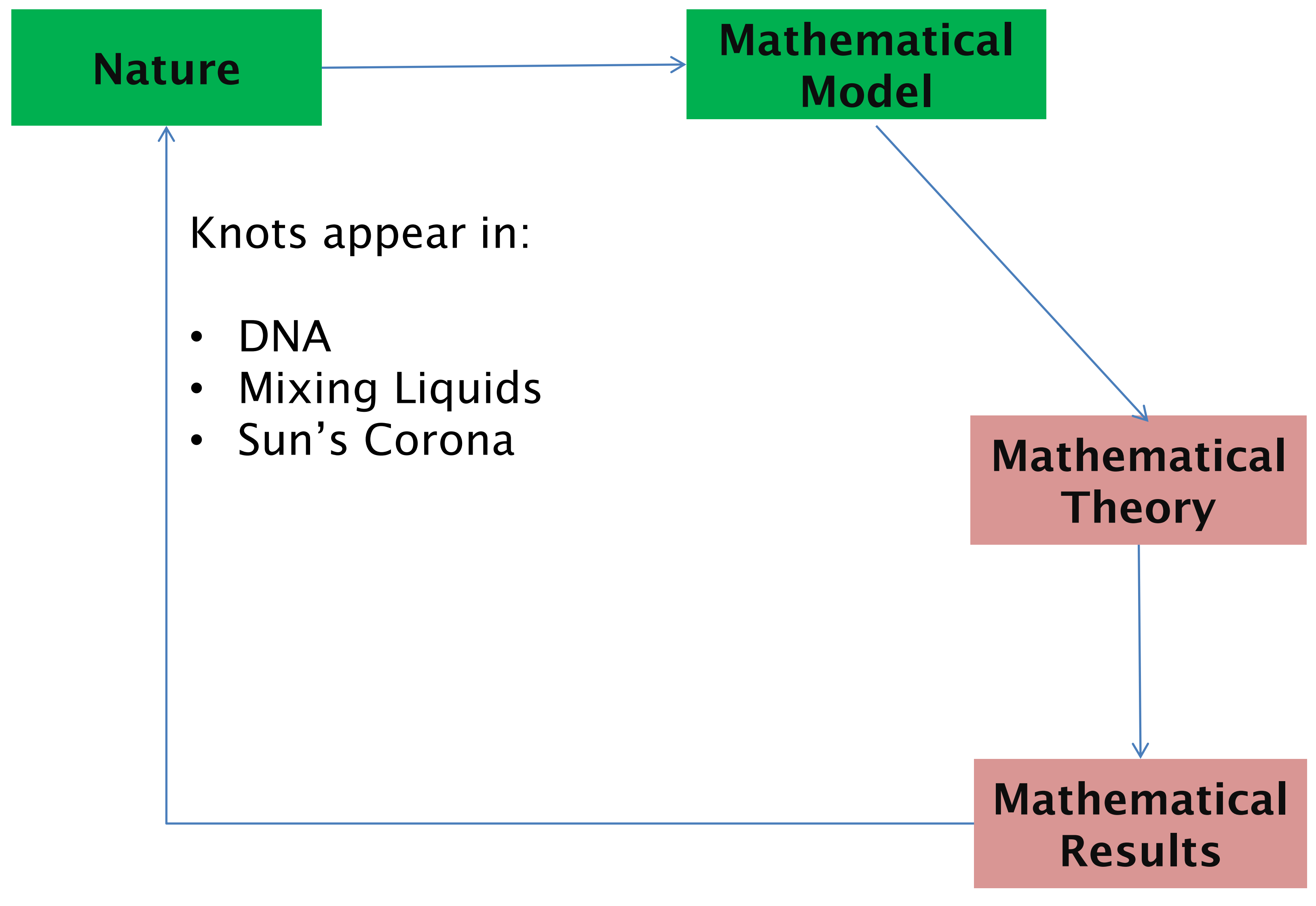



Nature
Mathematical Model
Mathematical Theory
Mathematical Results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Knots appear in:
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• Mixing Liquids
• Sun's Corona
Mathematical Theory
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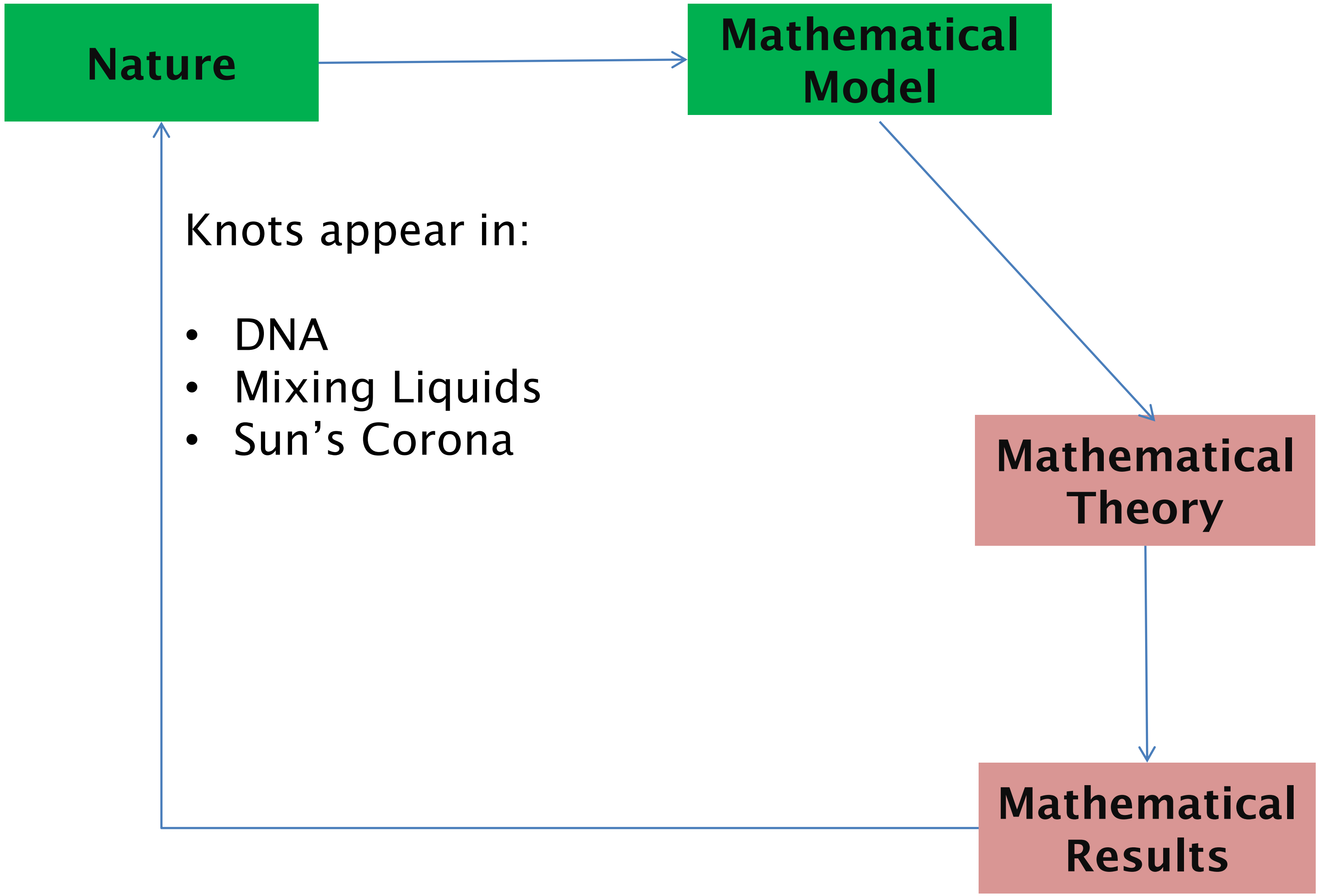

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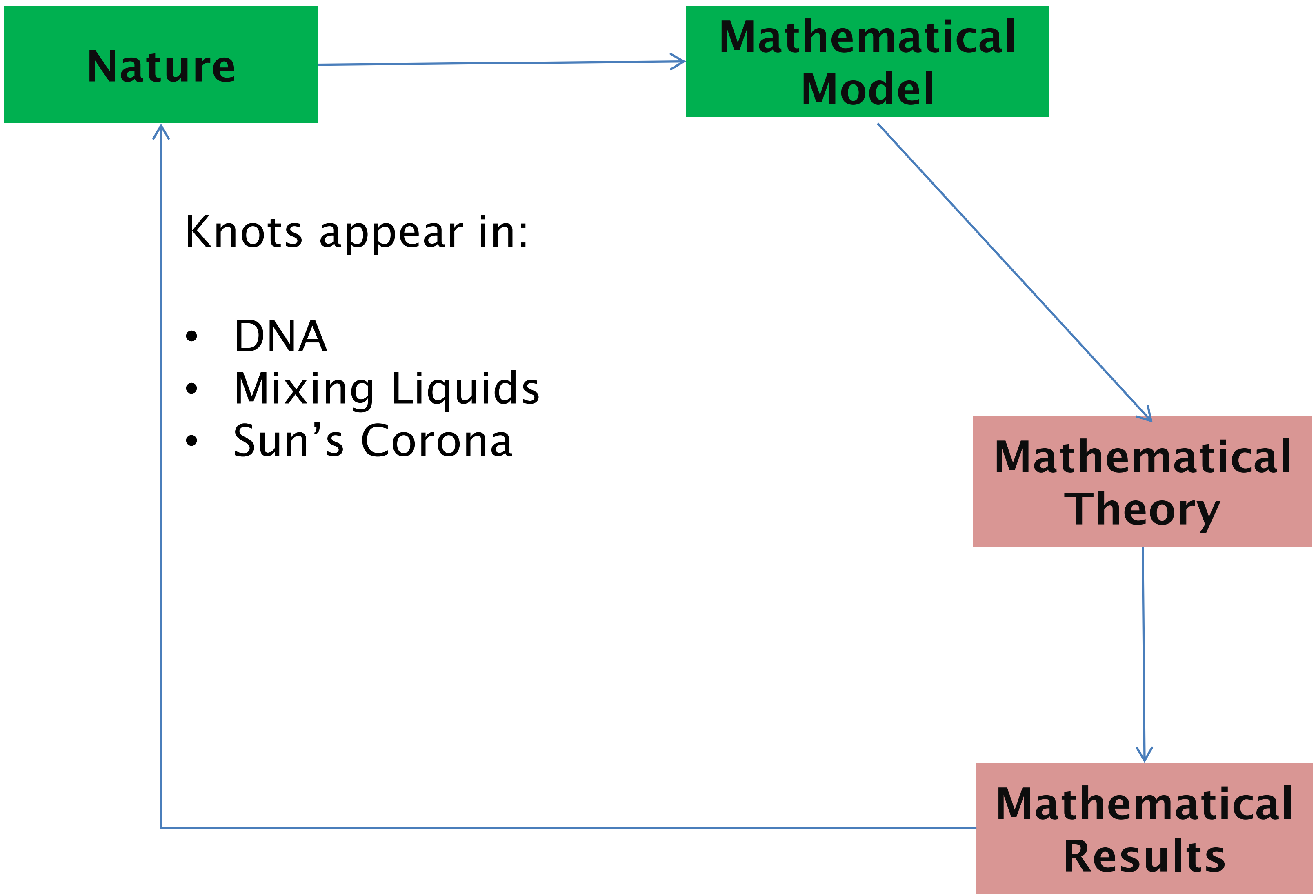
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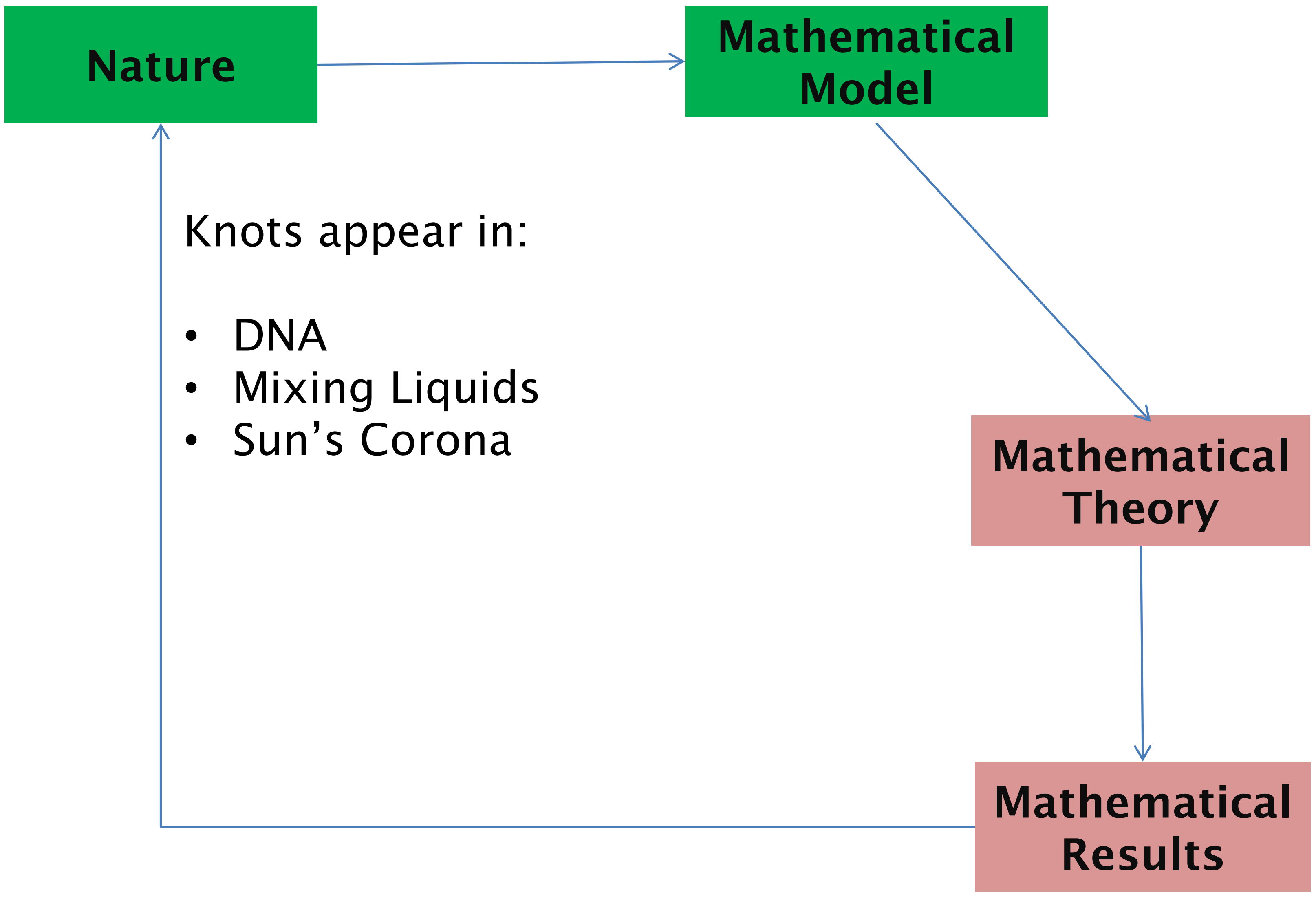
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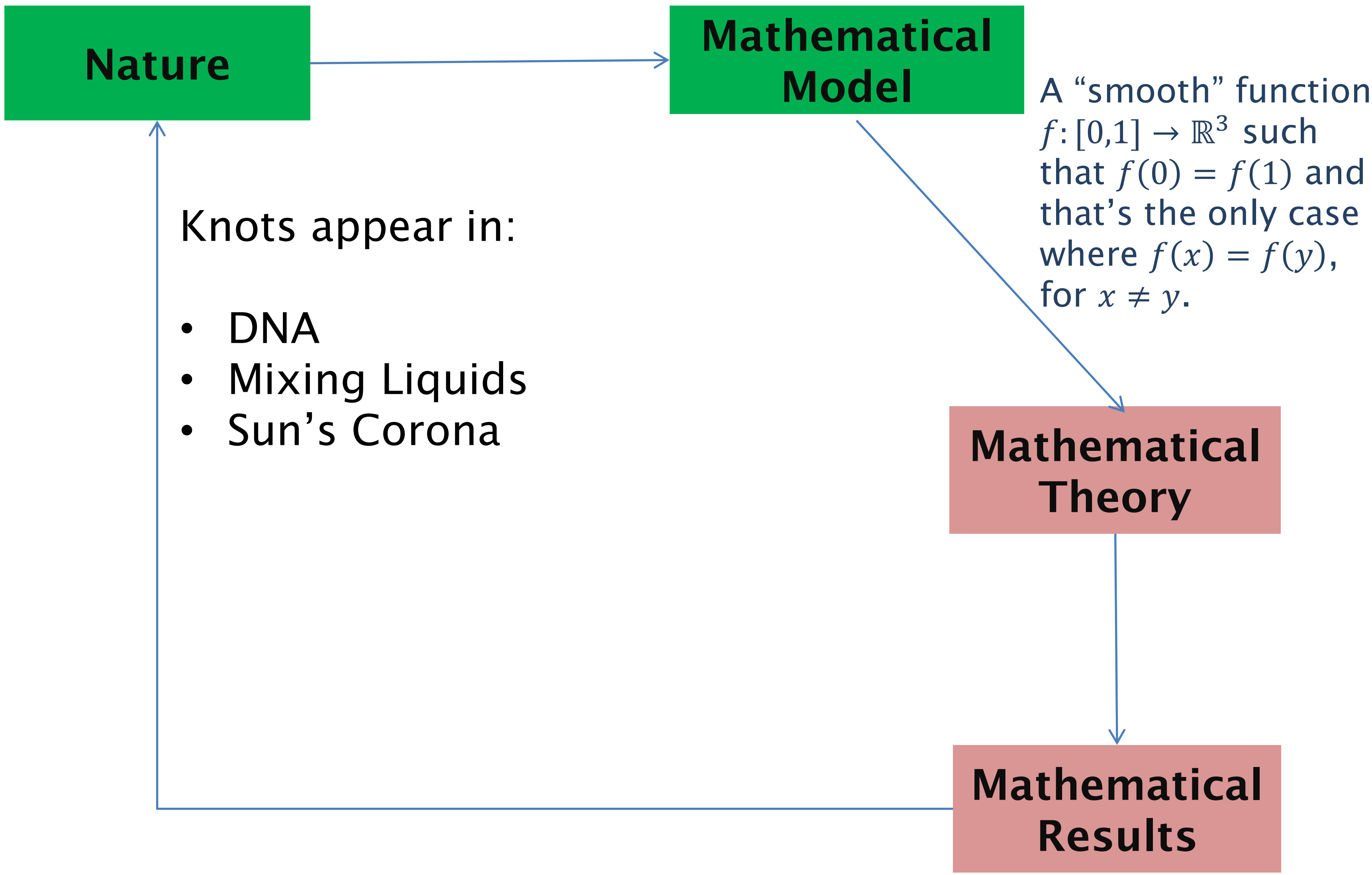
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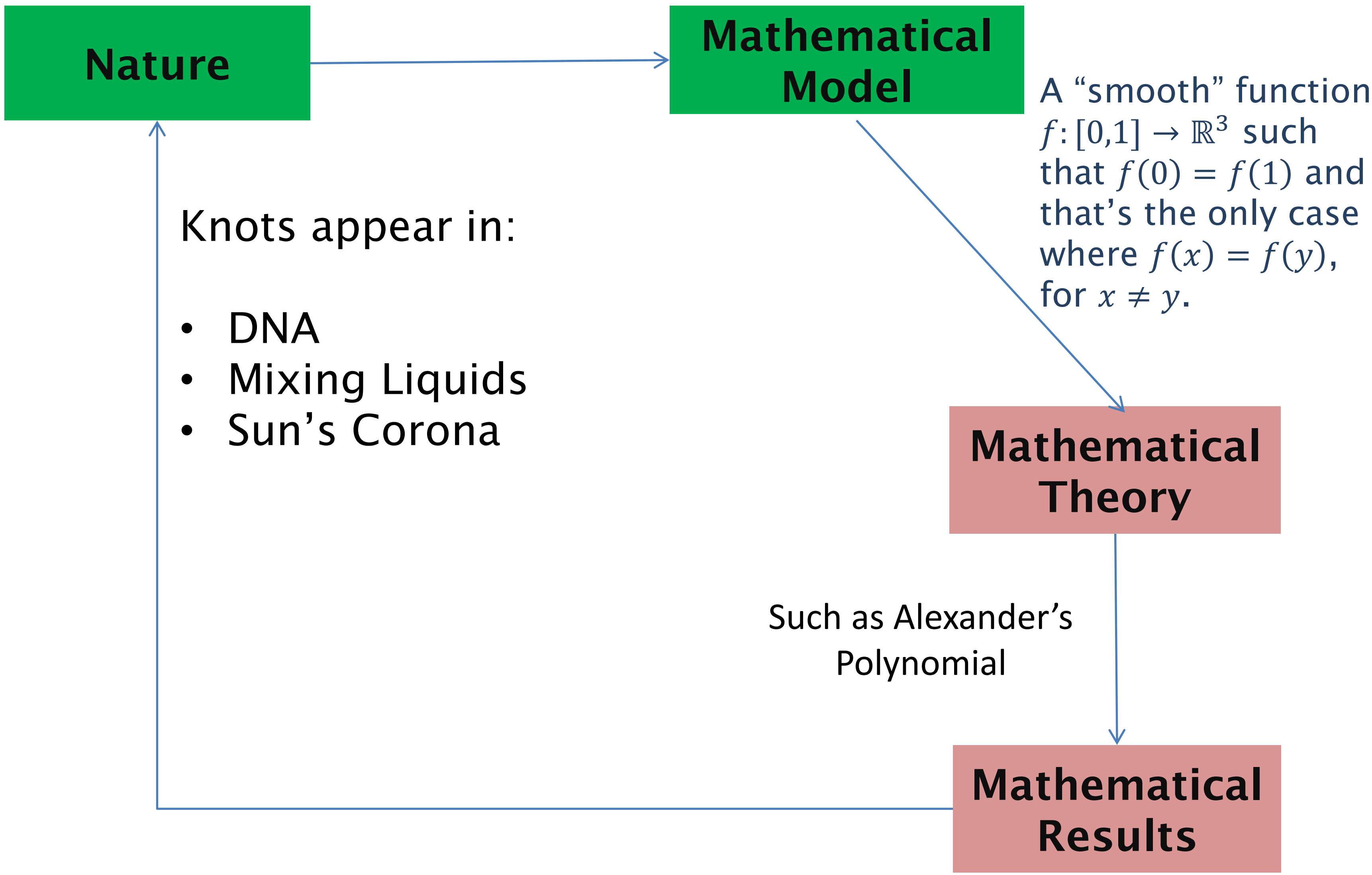
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Such as Alexander’s Polynomial
Mathematical Results

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Bibliography and Recommended Texts

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- *Colin C. Adams, **The Knot Book**,* American Mathematical Society,
  ISBN-13: 978-0821836781

- *Andrew Ranniki's Website has some great links*
- *Edward Long, **Topological Invariants of Knots: Three Routes to Alexander's Polynomial**, Manchester University, 2005*
- *Will Adkison, **An Overview of Knot Invariants***

Pictures used from

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